PME44 VIRTUAL

THE 44th CONFERENCE OF THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION

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Proceedings

of the 44th Conference of the International Group for the Psychology of Mathematics Education

VOLUME 1

Plenary Lecture, Working Groups, Seminar, National Presentation, Oral Communications, Poster Presentations, Colloquium

Editors:
Maitree Inprasitha, Narumon Changsri and Nisakorn Boonsena

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PREFACE

We are pleased to welcome you to PME 44. PME is one of the most important international conferences in mathematics education and draws educators, researchers, and mathematicians from all over the world. The PME 44 Virtual Conference is hosted by Khon Kaen University and technically assisted by Technion Israel Institute of Technology. The COVID-19 pandemic made massive changes in countries' economic, political, transport, communication, and education environment including the 44th PME Conference which was postponed from 2020. The PME International Committee / Board of Trustees decided against an on-site conference in 2021, in accordance with the Thailand team of PME 44 will therefore go completely online, hosted by the Technion - Israel Institute of Technology, Israel, and takes place by July 19-22, 2021. A national presentation of PME-related activities in Thailand is part of the conference program.

This is the first time such a conference is being held in Thailand together with CLMV (Cambodia, Laos, Myanmar, Vietnam) countries, where mathematics education is underrepresented in the community. Hence, this conference will provide chances to facilitate the activities and network associated with mathematics education in the region. Besides, we all know this pandemic has made significant impacts on every aspect of life and provides challenges for society, but the research production should not be stopped, and these studies needed an avenue for public presentation. In this line of reasoning, we have hosted the IGPME annual meetings for the consecutive year, July 21 to 22, 2020, and 19 to 22 July 2021, respectively by halting "on-site" activities and shift to a new paradigm that is fully online. Therefore, we would like to thank you for your support and opportunity were given to us twice.

"Mathematics Education in the 4th Industrial Revolution: Thinking Skills for the Future" has been chosen as the theme of the conference, which is very timely for this era. The theme offers opportunities to reflect on the importance of thinking skills using AI and Big Data as promoted by APEC to accelerate our movement for regional reform in education under the 4th industrial revolution. Computational Thinking and Statistical Thinking skills are the two essential competencies for Digital Society. For example, Computational Thinking is related to using AI and coding while Statistical Thinking is related to using Big Data. Therefore, Computational Thinking is mostly associated with computer science, and Statistical Thinking is mostly associated with statistics and probability on academic subjects. However, the way of thinking is not limited to be used in specific academic subjects such as informatics at the senior secondary school level but used in daily life.

For the PME 44 Thailand 2021, we have 661 participants from 55 different countries. We are particularly proud of broadening the base of participation in mathematics education research across the globe. The papers in the four proceedings are organized according to the type of presentation. Volume 1 contains the presentation of our Plenary Lectures, Plenary Panel, Working Group, the Seminar, National Presentation, the Oral Communication presentations, the Poster Presentations, the Colloquium. Volume 2 contains the Research Reports (A-G). Volume 3 contains Research Reports (H-R), and Volume 4 contains Research Reports (S-Z).

The organization of PME 44 is a collaborative effort involving staff of Center for Research in Mathematics Education (CRME), Centre of Excellence in Mathematics (CEM), Thailand

Society of Mathematics Education (TSMEd), Institute for Research and Development in Teaching Profession (IRDTP) for ASEAN Khon Kaen University, The Educational Foundation for Development of Thinking Skills (EDTS) and The Institute for the Promotion of Teaching Science and Technology (IPST). Moreover, all the members of the Local Organizing Committee are also supported by the International Program Committee. I acknowledge the support of all involved in making the conference possible. I thank each and every one of them for their efforts. Finally, I thank PME 44 participants for their contributions to this conference.

Thank you

Best regards

Associate Professor Dr. Maitree Inprasitha
PME 44 the Year 2021
Conference Chair

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VOLUME 1

PLENARY PANEL

WORKING GROUPS

CREATING CONSTRUCTIVE INTERFERENCE BETWEEN THE 4TH INDUSTRIAL REVOLUTION (+COVID 19) AND THE TEACHING AND LEARNING OF MATHEMATICS

Hee-chan Lew[1] and Anna Baccaglini-Frank[2]

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This paper presents the authors' argument in favor of the statement "The 4th Industrial Revolution will transform/disrupt the teaching and learning of mathematics", focus of the Plenary Panel at the Conference PME44.

THE 4TH INDUSTRIAL REVOLUTION AND THE TEACHING AND LEARNING OF MATHEMATICS

The 4th Industrial Revolution (4IR) refers to an industrial change that is based on a "virtual physical system" that can intelligently control objects through a high-speed network 5G. As stated by Klaus Schwab, Founder of the World Economic Forum, in the Encyclopedia Britannica online (2020): "The revolution heralds a series of social, political, cultural, and economic upheavals that will unfold over the 21st century. Building on the widespread availability of digital technologies that were the result of the Third Industrial, or Digital, Revolution, the Fourth Industrial Revolution will be driven largely by the convergence of digital, biological, and physical innovations." However, today, to discuss the impact of the 4th IR on the *teaching and learning of mathematics* (TLM) we have to consider the binomial "4IR+COVID 19". Indeed, the advent of COVID 19 is accelerating the development of high technology like Virtual Reality and Augmented Reality which makes our educational environments more connected dynamically through on-line tools and the internet. On the other hand, despite such high technology, schools shutdown and unsatisfactory online education could have (and in some cases has had) devastating effects on the TLM. To give a glimpse at what the "4IR+COVID 19" disruption has looked like at the beginning of the pandemic in Italy for the second author, we provide an excerpt from her personal diary from the end of March 2020.

"Because of the COVID 19 pandemic, on March 5, 2020, all schools in Italy have been closed to students till March 15th - the shutdown has been extended to April 3rd, and possibly it will be extended further [indeed it was...till September 2020]. Any form of assembly is prohibited and even leaving one's

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home unless strictly necessary is strongly discouraged. Teaching at all school levels in presence has instantly been suspended and transferred online (after a few weeks of "nothing"), with no shared methodology or support with the platforms. Like that, all of a sudden, the TLM has taken on monstrous forms in many cases: teachers upload piles of "exercises", some post hours of videos of themselves simplifying expressions or solving inequalities on a blackboard, others sign into their virtual classrooms, share the screen of a tablet, and start writing endless pages of equations without hardly taking the time to breathe. It is the death of the TLM. When the internet's bandwidth is enough to keep students and families "logged on" the challenge to try and "learn" any mathematics in these conditions - and in isolation from everyone except family members (when they are not positive) - is overwhelming. Many families "shut down", others cannot connect in the first place, others try keeping up by printing piles of worksheets and monitoring their children's work. It is a disaster. In many cases all the wealth of possibilities brought on by the 4IR is backfiring and leading to further frustration and isolation."

In this short paper we argue that there is no question whether "disruption" to the TLM has happened, but, depending on whether or not people have soft skills and willingness to keep on fighting and looking for ways to connect with and help as many students as possible, such a disruption can go in two opposite directions. We will talk about it through the metaphor of wave interference: *Destructive interference* between the 4IR+COVID 19 and the TLM leads to a descending spiral in which mathematics becomes completely obsolete and is perceived as useless in school and, more in general, in life; the opposite direction is represented by *constructive interference*.

Constructive interference is what we need to strive for, now more than ever! To do so, we must acknowledge that digital technology equipped with artificial intelligence (AI) is becoming more and more responsible for many human activities; that innovations at the intersection of biology and technology are likely to allow implants that enhance memory, among other things; that the notion of classroom in a single physical space with chairs, desks and blackboards is becoming only one of many possible (if so!) learning settings. So, we now need to develop different sets of skills like critical thinking, creative thinking, problem solving, as well as other soft skills such as communicating and team working. In order to create constructive interference between the 4th IR and the TLM, we argue that mathematics education should focus on and explore new and more effective ways to achieve goals, such as: appreciating mathematics as a tool for exploring nature and developing technology; communicating with machines; integrating mathematics learning environments inside and outside of the classroom (whatever *classroom* now means); appreciating statistics and ways of managing big data; promoting freedom and equity at the social level.

How can we go about doing this? Focusing specifically on the TLM, first of all we believe that constructive interference requires modernization of TLM for students to gain manpower to accelerate sustainable development of the 4IR, now that creative thinking is the most important, in contrast to the previous education model for training “good workers” with basic knowledge and skills that were necessary to manage mass production systems in the 1960s and 1970s (e.g., Robinson & Aronica, 2018). Such modernization must take into account a range of aspects, including: the overcoming of "encyclopedic knowledge" in favor of interdisciplinary connected learning; opportunities to personalize learning and make it more inclusive (not "the same" for all, but providing equal opportunities to all); shifting from the idea of a "curriculum to cover" towards interconnected "big ideas". The technological resources provided by the 4IR now make such a restructuring possible: it is up to us to make it happen. In the rest of this paper, we will focus on three aspects that, based on our own experiences as mathematics educators, we think should become central in the modernization process.

ASPECT 1: STATISTICAL THINKING AND INTERPRETING BIG DATA

Statistical thinking as an important tool for understanding uncertain phenomena involves procedures: To quantify data or information (describing), to identify a meaning of the quantity (making knowledge), to infer the whole from the partial (inferencing), to correctly make a decision based on inferencing (applying) (MOE, 2015). Statistical thinking in a digital society has a special meaning in that it provides ways to understand "big data", that is the huge amount of information with a wide variety of patterns and trends that is being generated constantly. The interest in big data is growing exponentially in today’s society because big data represents a paradigm shift in the ways that we understand and study our world. For example, Bakker and Wagner (2020) have stated: "The current situation asks for mathematical and statistical literacy of a large population, as the media use all kinds of representations and simulations to explain why the spread of the virus has to be slowed down, and why isolation is so important so as to keep numbers within the capacity of the health care system (“flatten the curve”)" (p. 3). So, one direction is making sense of things that happen around us, but another is machine learning (also see Aspect 3 of this paper). Here we focus on the importance of *interpreting big data to be able to design dynamic interactive learning environments* with intelligent support.

Advances in AI technology are enabling analyses of such data in ways that were unthinkable in the past. Big data in our society is an important asset for understanding the uncertainty of modern society and for making rational decisions as well as exploring social and natural phenomena of the future. For example, companies are striving to conduct marketing to suit individual tastes by analyzing customer-generated payment information, purchasing records and interests, while government agencies are analyzing huge amounts of information and utilizing it in various social areas to improve the efficiency of budget execution and the quality of public services.

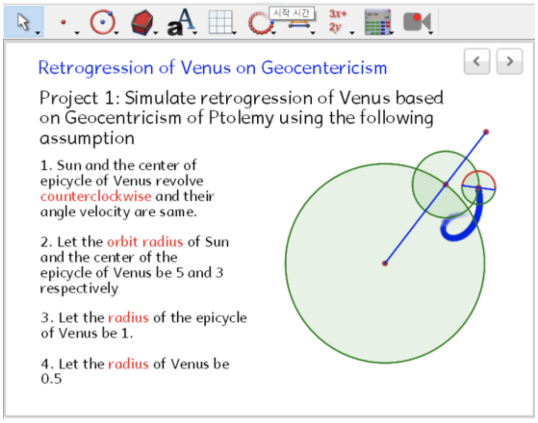
In the TLM we can achieve constructive interference between machines and humans by focusing on how to make rational decisions based on large scales of realistic data, delivered in real time through digital devices. Big data is characterized by 3+2 Vs: Volume, Variety, and Velocity, together with Veracity and Value of data. Veracity is a feature that describes how data can be reliable. We need to educate students to understand that data from social networks cannot be considered accurate because it does not always provide reliable evidence. For example, students can be led to explore ways to extract accurate data, "washing out" the inaccuracies and errors; and students can be invited to discuss the value of data. Through big data processing using high level ICT technology, it is now possible to collect and analyze data in real time and present analytical results in various forms. The ability to find trends hidden in big data and communicate using visual information so that the analyzed results can be easily understood is essential for students who will continue to lead the information age.

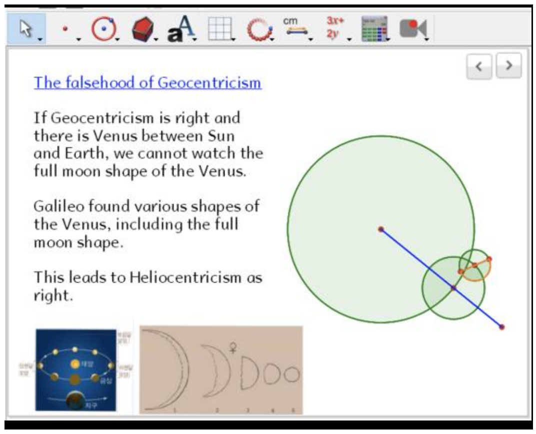
ASPECT 2: EXPRESSING IDEAS IN DIFFERENT "LANGUAGES"

Today many digital artifacts are available at the swipe of a finger, on many kinds of portable devices; moreover, we have experienced a convergence of physical and digital innovations, 3D printing and new block-based programming languages that can provide engaging and motivating contexts for learning. Interacting with these artifacts, students can be drawn into mathematical discourse, while simultaneously learning new languages that allow them to interact with machines. We see these forms of interaction as potentially beneficial for fostering constructive interference for at least two reasons: First, they can help students learn human-machine communication by providing immersive experiences; second, such interactions occur in different "languages" that, next to the formal language of mathematics, provide multiple means for students to communicate with each other and with the teacher in meaningful and inclusive ways.

Dynamic interactive textbook representations

Figures 1a,b show pages of a dynamic interactive textbook (Lew, 2016) developed using the Cabri applet, where "planet movement" is used in the "Circle" unit for the 9th grade in junior high school. Here students can interact with a model of the retrogression of Venus based on Geocentricism developed by Ptolemy in the 2nd century and they can use the situated context of the applet to reason about and discuss how Geocentricism was rejected by Galileo (Lew, 2016). Research has shown how through dynamic interactive applets, we can offer students the opportunity to manipulate different representations of functions, making conjectures about how functions describe *change*, and how this can be represented, talked about, and hence thought of within different semiotic registers of representation (e.g., Lew, 2016; Antonini, Baccaglini-Frank & Lisarelli, 2020). These dynamic and interactive representations of a function, in addition to being quite engaging and motivational for students, provide easy entries into situated forms of discourse that, with the help of the teacher, can evolve into more formal mathematical discourse (e.g., Antonini et al., 2020; Baccaglini-Frank, 2021).

a) 

b) 

Figures 1a,b: Simulation of the retrogression of Venus according to Geocentricism.

In general, research has suggested that dynamic interactive applets and textbooks have a positive impact on classroom practice, providing an environment for students to communicate in new, meaningful, and inclusive ways (e.g., Doorman, Drijvers, Gravemeijer, Boon & Reed, 2012; Lew, 2016; Lew, 2020).

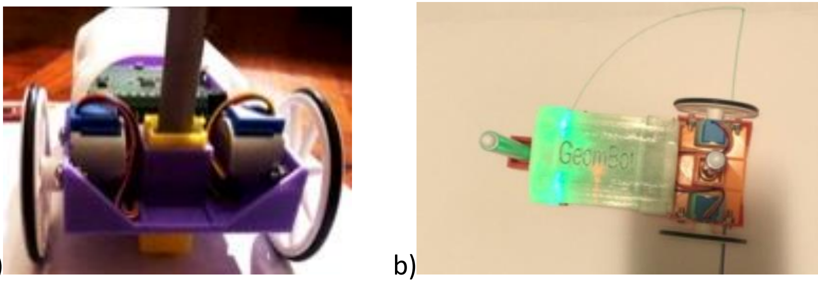
Geometry through the language of the Geombot

While most of the activities with robots proposed in classrooms currently aim at introducing "programming" or "coding" per se - when they have any explicit educational aim at all - to foster constructive interference between the TLM and the 4IR, we build on a research tradition (initiated in the 1980s) that has focused on the integration of programming into mathematical learning (e.g., Papert, 1980). In this perspective coding becomes a language for expressing mathematical ideas, such as algorithms and equations, but also geometrical properties of a figure (for example, to be drawn out on paper by a robot), that can be learned from a very young age.

This has been a direction of research over the last few years of the second author, who designed a drawing robot, the GeomBot, for primary school students. The GeomBot combines the well-known strengths and opportunities offered by Papert's original robotic drawing-turtle with those of the block-based programming language Scratch; and it builds on the convergence of physical and digital affordances. Most physical parts of the GeomBot are designed using

3D modelling software (SketchUp and OpenSCAD) and printed using a 3D printer. The GeomBot can hold a marker between its wheels that draws out its path as it moves on a sheet of paper on the floor, as well as a marker at the front, to highlight rotations (Figures 2a,b).

a) b)



Figures 2*:* a) back view of the GeomBot; b) top view of the GeomBot.

To "talk to" the GeomBot the student needs to construct a sequence of command blocks designed in Snap!. The sequence is then transmitted to the robot wirelessly, once it is clicked. Although these blocks are virtual objects that "live" on a screen (touchscreen of an interactive white board, tablet, or computer screen), they are concrete enough to be accessible to and shared by all students as they engage in planning a drawing, and in programming and debugging a sequence based on the physical feedback given by the robot. A recent study has shown how it is possible for in-service teachers involved in professional development for 9-months with the GeomBot to significantly shift their perspectives towards the teaching and learning of geometry at many different levels, including understanding of mathematical contents, using different languages, inclusive mathematics education and overcoming a fear towards technology (Baccaglini-Frank, Santi, Del Zozzo & Frank, 2020).

ASPECT 3: UNDERSTANDING AI

Artificial Intelligence (AI) is a basis of highly sophisticated technologies driving the 4IR. AI can be looked as machine systems, or as a problem-solving strategy. As a machine system, AI aims at implementing a variety of recognition, thinking, and learning processes that before had been performed by only human intelligence. AI aims at modeling human intelligence. For example, an autonomous driving vehicle is a car equipped with AI which can recognize various environmental information and make a decision for driving safely. As a problem-solving strategy, AI can make a computer "think" efficiently. For example, machine learning and deep learning allow machines to analyze and process data and information that humans need. To pursue constructive interference, AI should gradually but systematically address in schools.

Indeed, education poses new challenges. UNESCO (2019) recommends governments to consider planning AI in education policies and to develop appropriate capacity-building programs for teachers. In Korea constructive interference is already taking place: curriculum for AI education and various high school AI textbooks are being developed and they will soon be offered through public education (MOE, 2020; Lee, Im, Jang, Song, Kong & Park, 2020; Youn, Kim, Nam, Choi, Jung & Kim, 2020). Furthermore, in 2019 the South Korean government decided to train a set of so-called “AI teachers” who are responsible for fostering new human resources to lead the new era of the 4IR. Each year for 5 years, 1,000 AI teachers will receive master's degrees through three years of in-service training, beginning in the second semester of 2020.

What is the most important area to be taught in mathematics education regarding AI for constructive interference to occur? Following Isoda (2021), school mathematics needs to foster computational thinking, since it is a critical cognitive skill that should be used efficiently to solve problems using computers and digital tools. Computational thinking should be included in the design of modernized mathematics curriculum in the digital society because it is powerful for students to formulate problems in a way that enables them to use a computer, to simulate the most efficient and effective solutions, to generalize the solutions to a wide variety of problems.

Specifically, students should learn and experience three core computational thinking processes which have been introduced as powerful tools to solve various problems in the history of AI (Araya et al., 2019): *algorithmic thinking*, *computational modeling*, and *machine learning*. Focus on these processes can help achieve constructive interference: their learning can foster meaningful problem-solving activity, while simultaneously provide an engaging application of mathematical knowledge.

CONCLUSION

Although mathematics has been at the heart of brilliant civilizations for centuries, today it is undervalued, and students often regard it as an irrelevant and meaningless subject. We believe mathematics to be one of the essential subjects in the twenty-first century and more central than ever in the midst of the 4th IR+COVID 19. We have highlighted aspects of the 4th IR upon which technological environments, tools and activities in mathematics education can be based to achieve constructive interference between the TLM and the 4th IR+COVID. Indeed, we believe it to be possible–and in some places this is already occurring– to take advantage of our societies' technological advances to design environments, tools and activities to help students appreciate mathematics as a tool for exploring nature and developing technology, communicating with machines, integrating mathematics learning environments inside and outside of the classroom, appreciating statistics and ways of managing big data. If the TLM can successfully embrace the kinds of modernization we propose, students will develop creativity and other key skills for their future, recognizing mathematics not as fragmentary knowledge, but as an interconnected body of knowledge through which to understand the world around them. However, such a modernization of the TLM is by no means automatic, nor does its application in some places imply its spreading worldwide. So, as a society, we must continue to strive together to propagate constructive interference.

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