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ABOUT THESE PROCEEDINGS

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SHIFTS FROM TEACHING MATHEMATICS *WITH* TECHNOLOGY TO TEACHING MATHEMATICS *THROUGH* TECHNOLOGY: A FOCUS ON MATHEMATICAL DISCUSSION

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Mathematical discussion plays a key role in teaching-learning processes, and it has been mostly studied when implemented in physical classroom contexts. However, during the lockdowns imposed by the Italian government (and many other governments worldwide) because of the COVID-19 pandemic, teaching had to be performed exclusively online. Therefore, also mathematical discussions, when implemented, had to be carried out through digital technology. In this paper, using the perspective offered by the Theory of Semiotic Mediation on collective mathematical discussions, I analyze an online mathematical discussion on division algorithms in a 6th-grade class. The analyses point out significant differences in how the actions were implemented online and in their effect in the discussion, especially on the emergence and elaboration of signs.

Keywords: Online mathematical discussion, signs, teacher actions, Theory of Semiotic Mediation.

HOW MATHEMATICS CAME TO BE TAUGHT THROUGH TECHNOLOGY IN ITALIAN CLASSROOMS: THE COVID-19 CRISIS

To contextualize this study, I start by citing a very representative quote from a recent paper by Ramploud et al. (2021) describing the general feeling of most (if not all) Italian teachers during the COVID-19 crisis:

> […] it was a violent and uncontrolled cultural change that teachers had to face at a moment of extreme isolation, since communication with and among students was to be necessarily combined with technological tools, changing the nature of communication […] (Ramploud et al., 2021, p. 5)

The authors describe the situation as a *crisis*, referring to Yerushalmi (2007):

> Crisis situations, as I am defining them, occur during ordinary life occurrences when one's personal mechanisms for organizing experience cease to function. […] Often the primary experience is one of internal imbalance and lack of control over one's life. (Yerushalmi, 2007, pp. 359–360)

Although, unfortunately, this was the situation within which most Italian teachers felt trapped, they (at least the teachers from our research-action groups [1]) continued doing their best to support the emotional well-being and mathematical learning of their students. In this crisis situation, many students did not dispose of adequate devices and internet connections, so most of the teachers decided not to propose activities with digital artefacts during the regular online lessons. Such a decision was in contrast with what I and other researchers involved in the research-action projects tried to propose and had hoped to be able to study. However, many of the teachers in our groups did try to promote mathematical discussions online. This is the situation that I use in this paper to exemplify teaching *through* technology. In particular, I will highlight shifts in the practice of mathematical discussion when it is carried out online as opposed to in physical classrooms.

Conceptualizing the use of digital technology in the "crisis" context and my objective.

The didactical tetrahedron (Ruthven, 2012) foresees two main roles of technology: one describes digital technologies in their mediational role with respect to specific mathematical content, while the other is described by Ruthven (2012) as follows:

> Another line of development in the educational use of digital technologies has sought to update and enhance the basic infrastructure that supports classroom communication between teacher and students, and assists their use of content-related resources within and beyond the classroom. (Ruthven, 2012, p. 628).

Because of the data that I was able to collect and analyze prior to this talk, I will focus on lessons in which digital technologies are framed within this second line of development. Moreover, this will allow me to focus specifically on shifts that occur due to the different form of communication (online as opposed to "in presence") between teachers and students, avoiding overlap between the two lines of development. Within this second line, we can talk about way in which teachers use digital technology in terms of "schemes", as discussed in the Theory of Instrumental Genesis (Monaghan et al., 2016).

Within this perspective, our teachers were faced with the task of communicating with their students, to teach them mathematics, and to accomplish such a task they made use of the (digital) tools of Google Meet and Jamboard. So, studying how mathematical discussions are carried out through technology becomes a matter of inferring teachers' schemes, by looking at techniques (the visible parts of such schemes) used to communicate with digital technology during such online discussions.

Objective of this Paper

Specifically, I will focus on the online implementation in Meet and Jamboard of the typical actions (see the following section) used by the teacher in a mathematical discussion, which I expect to be qualitatively the same online and in presence. I will pay particular attention to how the teachers use this digital technology to foster the emergence and elaboration of *signs* in such a discussion. An underlying hypothesis is that there are differences in how the actions are implemented online and in their effect in the discussion, especially on the emergence and elaboration of signs.

SIGNS AND MATHEMATICAL DISCUSSION FROM THE PERSPECTIVE OF THE THEORY OF SEMIOTIC MEDIATION

The Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008) was developed to analyze the relationship between students' accomplishment of a task through an artefact and their mathematics learning, precisely addressing the issue of how students can become aware of the meanings stemming from the activity with an artefact to accomplish a task, and of how such meanings can evolve towards target mathematical ones. In this section, I introduce *collective mathematical discussions*, as they are conceptualized within the Theory of Semiotic Mediation, focusing specifically on the theorized teacher actions within such discussions according to the goal of promoting the evolution of signs. I will use the notion of *sign* in a broad sense, including any kind of perceivable spatio-temporal entities which might be uttered, spoken, written, drawn, encoded electronically, or in general used by someone to express some meaning.

Collective Mathematical Discussion

Classroom discussion to promote mathematical learning has been conceptualized in different ways (e.g., Michaels & O'Connor, 2015; Stein et al., 2008). Here I refer to *collective mathematical discussion* as introduced by Bartolini Bussi and Mariotti (Bartolini Bussi, 1998; Bartolini Bussi & Mariotti, 2008); it is part of each didactic cycle, and it is a kind of activity involving the whole class:

various solutions are discussed collectively, students' situated signs (with personal meanings) are collectively analyzed, commented, and elaborated. Students' interventions are coordinated by the teacher with the goal of generalizing the situated meanings, emerging from the activities with the artefact, and moving them towards mathematical meanings. (e.g., Bartolini Bussi & Mariotti, 2008; Mariotti & Maffia, 2018).

Teachers' Actions in Collective Mathematical Discussions

The study of teachers' actions aimed at fostering the production and development of signs during mathematical discussions has led to a classification of such actions (Mariotti, 2009) that Mariotti and Maracci (2010) have also described in terms of schemes. Below are the four types of actions conceptualized in these terms.

Back to the task: the class of situations characterized by the need of promoting the students' production of signs related to the actual use of the artefact for accomplishing a given task.

Focalize on certain aspects of the use of the artefact: the class of situations when the discussion has led to the emergence and sharing of a rich net of signs related to the use of the artefact and there is the need of selecting the pertinent aspects of their shared meanings in respect to the development of the mathematical signs that constitute the final education goal.

Ask for a synthesis: the class of situations when the discussion has led to the emergence of shared and stable signs condensing the key aspects of the common experience with the artefact, and there is the need of generalizing and decontextualizing the meanings that emerged.

Provide a synthesis: the class of situations, when the discussion has led to the de-contextualization and generalization of meanings form the context of use of the artefact towards the context of mathematics, and there is the need of ratifying the acceptability and the status of a sign within the mathematical context.

Application of this Theoretical Perspective to the Study in Focus

The teachers in this study were aware of the four types of actions described above, and they were used to enacting them during collective mathematical discussions conducted in presence in their classrooms. However, the teachers had not conducted discussions completely online before.

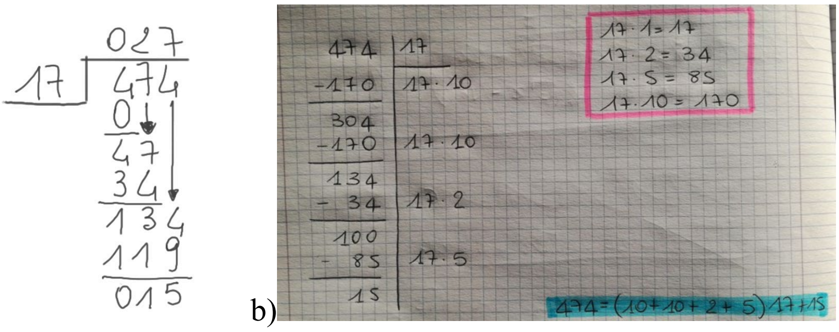
A slightly delicate issue is that of the artefact around which the discussion is centered, according to the didactic cycle. In many studies in the literature, including most of my own (e.g., Antonini et al., 2020; Baccaglini-Frank & Mariotti, 2010; Baccaglini-Frank, 2019, 2021), the artefact is a digital one. Such studies would, therefore, fall into the first role of technology described by Ruthven, that I introduced above. However, in this paper, the artefacts around which the discussion is promoted are two division algorithms [2] (Lisarelli et al., 2021). Therefore, the signs I will be analyzing refer to these algorithms and mathematical meanings behind them, such as division, place value of digits in numbers written in base ten, or powers of ten.

However, here I will not be looking at how digital technology mediates mathematics, but at how it mediates the communication *about* mathematics between the teachers and the students, which is consistent with Ruthven's second role of technology.

PARTICIPANTS, MATHEMATICAL CONTEXT AND DATA COLLECTION

The mathematical discussion in this paper is part of data collected during the Italian lockdown in March-April, 2020 from classes of the teachers of our research-action group [1]. Specifically, the discussion occurred online in one of the two subgroups that a 6th-grade class was divided into for some of the online activities (as described in Lisarelli et al., 2021). The online discussion was conducted by F., the main teacher, together with G., a researcher, who had been involved in the design and implementation of the teaching sequence on division algorithms.

The class was part of an English school in Northern Italy, in which mathematics had been taught in English until the end of grade 5, while in grade 6 it was taught in Italian. In primary school the students had been taught an algorithm they called “DMSB”. The acronym stands for the steps: divide, multiply, subtract, bring down (Figure 1a). Prior to the lesson in focus, the class had been introduced to what was referred to as the “Canadian” algorithm (also known as “Nuffield”, see Figure 1b) [3], through the escamotage of a letter from an imaginary Canadian student Nadège. Prior to this lesson, the students had used both algorithms to calculate quotient and remainder of various divisions, but they had never been asked to compare these two algorithms.



a) b)

Figure 1. a) Divide, Multiply, Subtract, Bring down (DMSB) algorithm; b) “Canadian” (non-optimized) or “Nuffield” division algorithm

During the online lessons (including the one in focus), the students were asked by their teacher F. to keep their webcams and microphones off unless she asked them to activate the microphone, while the webcams had to stay off. The students had free access to the online chat in Meet. The mathematical discussion occurred in Italian, and it was transcribed in Italian and then translated into English.

Setting the Scene of the Lesson with Respect to the Didactic Cycle

F. and G. had assigned the division $395{:}\,16$ to be carried out with the DMSB and with the Canadian algorithms as homework. They had collected the students’ work and transcribed some of the solutions to use at the beginning of the lesson, before launching the collective discussion. G. was in control of the Jamboard and, together with the class teacher, F., guided the discussion. F. had announced, at the beginning of the lesson, the objective in the following terms: “Let’s be mathematicians and try to compare these two algorithms and discover why they produce the same quotient and remainder.”

SPECIFIC FINDINGS FROM THE ANALYSES OF EXCERPTS FROM THE ONLINE COLLECTIVE MATHEMATICAL DISCUSSION IN A 6TH GRADE CLASS

For space limitations, I will not include here the complete transcriptions of the excerpts I report on, some of which are included as subtitles of the video clips shown during my talk (visible here: https://youtu.be/WqEQHtP5_fs). Here I will only list some of the findings from such analyses, organizing them around the main tools used, and trying to follow the discussion’s temporal evolution.

Using the Chat in Google Meet to See Students’ Signs and as a Window onto the Class

Both F. and G. frequently open the chat and scroll down, in search for students’ signs to pick up and use in their actions. Especially at the beginning of the discussion, they use the students’ signs written in words for focalization actions. We note that signs in chat are only expressed in a single modality, as written words, unlike in "in presence" classrooms, in which F. would usually capture gestures and oral expressions together with written signs on the students' worksheets or notebooks. So, this online action appears to dramatically narrow down students' initial modes of sharing signs they have produced. When promoting online focalization actions, F. and G. tend to call on students who have produced certain signs to turn on their microphone and explain, adding another mode of production and potentially new signs to the semiotic bundles (Arzarello, 2006) produced.

The teachers also use the online chat to get an overview of how the class is following. For example, after providing a synthesis of what Ka. has noticed, Figure 2a shows F. asking whether the class agrees with Ka. and immediately after, she and G. scroll through the chat. Many students write "yes" or words suggesting approval. However, F. seems to use that chat also to see who remains silent, that is, "invisible", to later call on them and ask them to participate.

a) b)

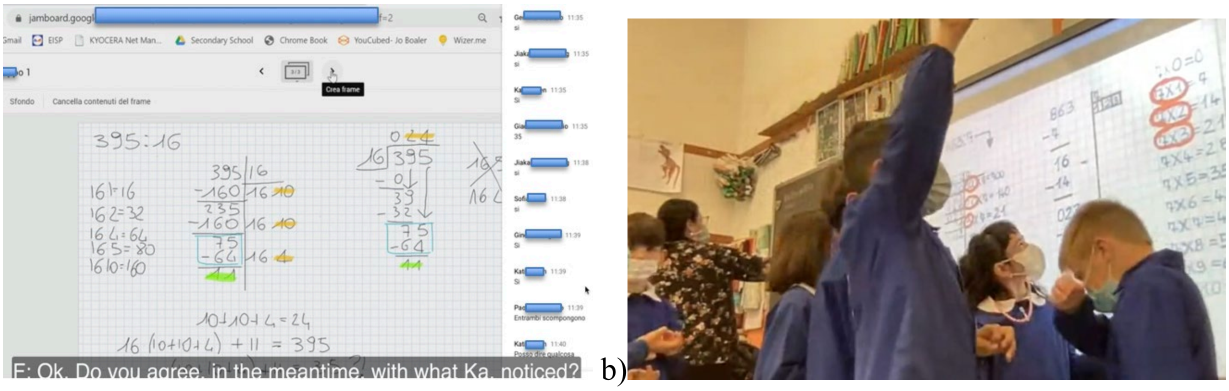


Figure 2. a) The teachers using the chat as a window onto their online class; b) visual glimpse at student participation in a physical classroom, in response to a similar question by the teacher

This way of using the chat seems to be an online version of what visual and acoustic signs convey in physical classrooms. Figure 2b shows students in a physical classroom responding to a similar intervention of their teacher in a mathematical discussion. However, unlike in a physical classroom, the chat allows to "see behind" students' raised hands, because frequently students write brief answers to questions posed by other classmates or by the teacher. In the discussion analyzed here, a student, Al., referring to the "75" that appears in both algorithms, disagrees with a classmate who says: "they are the same" and asks to speak, writing in the chat: "[but] coming from two different sums".

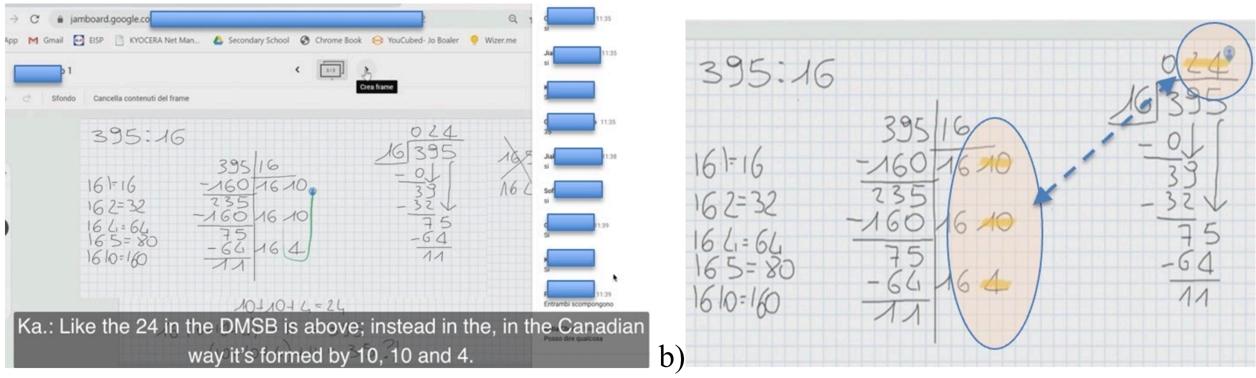
Using Jamboard for "Online Gesturing" in Coordination with Students' Speech through the Mic

Initially, G., who is in control of the shared board, tries to coordinate her production of signs Ka.'s words as she speaks (without being seen or being able to act on the Jamboard). For example, in Figure 3a, G. highlights with the laser tool the numbers Ka. is referring to. This technique, a sort of "online gesturing", serves to bring attention to certain graphical signs, part of the bundle with the oral signs, as well as to confirm to Ka. that she is being understood. Notice that, in a physical classroom, these or similar gestures accompanying verbal utterances would all be produced by the student.

F. suggests a modification of G.'s technique to produce more permanent signs on the board, as shown in Figure 3b.

Ka.: Like the 24 in the DMSB is above; instead, in the, in the Canadian way it's formed by 10, 10 and 4. (Figure 2a)

F.:	Uhm, can you circle them, G., with the same color using the highlight tool, maybe? ... Leave them colored.



a) b)

Figure 3. a) G. uses the laser tool as Ka. speaks; b) correspondence between signs described by Ka. and highlighted graphically by G.

Now Ka.'s verbal utterances are completed with corresponding graphical signs, that are highlighted to make the correspondence explicit. The correspondence between the 10, 10 and 4 and the 24 shown in Figure 2b is expressed orally, graphically, and through G.'s online gesturing. So the set of signs in focus is produced jointly through a collaboration between Ka. and G.. In this joint effort, the actors seem to have different, though harmonious intentions: Ka. wants to explain the correspondence between signs she has perceived but that she can express only verbally, while G. works hard to interpret Ka.'s words, in order to "see through her eyes" and show Ka. she is being understood, while at the same time enriching the set of signs with graphical components in the hope that other students will see the same connections that Ka. is describing. This coordination effort can be seen as a focalization action.

Indeed, G. notices Al.'s comment, finds it relevant in the orchestration of the discussion, and immediately uses it in a focalization action.

G.:	What does "broken into two different sums" mean?
Al.:	The two...I mean in the first one there is 235 minus 160... and, eh, and in the one on the right there is 39 minus 32.
F. & G.:	Uhm! Uhm!
G:	What do you see as similar? What does "broken un into two different sums" mean?
Al.:	No, in the sense that they derive from two different sums, the same subtraction.
F:	It means that the 75 comes out from two different processes, Al. is saying.
G:	So Al., are you suggesting that this package―I don't know if I'm going too far beyond or if it's what you meant―should somehow correspond to this little package here? (Figure 4)

In the excerpt above, notice how Al. uses a sort of verbal enaction of a pointing gesture ("on the left/right"), elaborating on the sign "different sums", which he then refers to as "the same subtraction". F. elaborates on Al.'s sign, restating it as "different processes". This builds up to a key moment in the discussion, at which G. expresses the sign in terms of "packages" that she interprets, as shown in Figure 4. Such an interpretation does not seem to be completely coherent with Al.'s meaning.

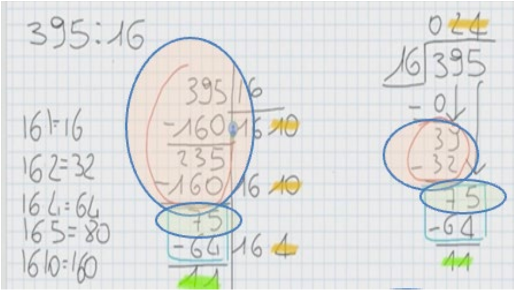


Figure 4. G. highlights the “packages” that she interprets Al. sees as corresponding in the production of the 75s in the two algorithms

G.’s focalization action includes a strong interpretation of Al.’s sign: while the student’s intervention refers to the two 75s “coming from different sums/subtractions”, G. uses oral and graphical marks to bring the class’ attention to “39-32” in the DMSB and to longer sequence in the Canadian division, adding the word “little package(s)” in correspondence to her signs (Figure 4). Moreover, Figure 4, together with Al. and G.’s verbal utterances, illustrates how different components of the semiotic bundle are produced and constantly re-elaborated by the actors, in this case, the student and the teacher, G.. This re-elaboration of the signs produced by the student seems to go far beyond his intended meanings: indeed F. will then spend most of the lesson trying to bridge the gap between the meanings at play, created by G.’s misinterpretation. In F.’s physical classroom, even when the class was co-conducted with G., we have not hardly ever witnessed such a discrepancy between a student’s intended meanings and those in the teacher’s interpretation of the shared signs. I expect this to be the case because, in the physical classroom, signs are elaborated more gradually by other students as opposed to by the teacher.

Using Speech through the Microphone Uncoordinated with Actions with any Other Tools

A standard practice in F.’s class is to have students “ask each other” when they do not understand something. During the online lesson, F. tries to use the same practice, but mediating students’ interactions much more, because of her rules on taking turns speaking by turning on the microphone, only when asked. Towards the end of the discussion, F. uses such a practice to ask Jia. to explain to Gem. something that had been shared by other students: why in the DMSB there is a “39-32” that gives 75 (not 7). This is a kind of ask for a synthesis action. F. rapidly follows up on such an action providing a synthesis, herself, in which she further elaborates on the signs produced by the class.

Jia.: Instead, the 39 is simply without the 5 that then in brought down to the 7.

F.: Jia. is saying that 39 is actually a 395,... the 5 stayed up top.

F.: Then, instead she says that that 32 isn’t in fact exactly a 32, but it’s 320, there is a zero hiding away a bit there. And that 320 is exactly the sum of the two 160[s] instead in the Canadian [algorithm].

Focusing on the signs in this excerpt, there are at least two sets of signs that are put in relation with one another: one that includes the “bring down” in the DMSB, and one including “hidden” digits (these two sets are highlighted graphically in Figure 5). As shown previously, here, too, different components of the sets of signs are produced and managed across the actors: Jia., like her classmates who spoke before her, explains only orally, without the possibility of marking the board or even pointing to signs on the board; F. repeats and rephrases some of the words uttered by Jia., but because she is not in control of the Jamboard, her signs are also expressed in a single modality, though they refer to numbers written on the board.

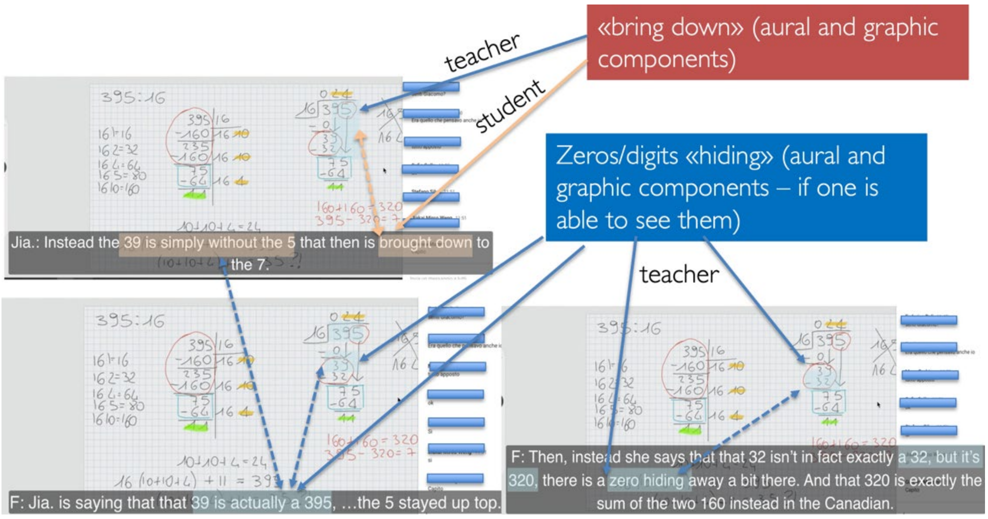


Figure 5. Two sets of signs co-produced by the class and the teachers

Unlike what she did with students, G., is not supporting F.'s synthesis with any online gesturing. Therefore, in F.'s synthesis the connections between visual and verbal components of the sets of signs are left implicit. I tried to highlight some of these connections with the additional markings I made in Figure 5. This surely shows that the ways in which signs are produced and shared on online discussions like this one is quite different than those that take place in physical classrooms. Possibly, a lack of explicit connections, like in the case shown above, can be stimulating for some students, who in this way are not shown everything, but they need to actively follow others' words to visually navigate the page. However, this set up may be problematic for some students, as it surely creates a heavier cognitive burden for them to carry. At the moment, I do not have enough data on the effects of this sort of online teaching practice to discuss it any further.

Using the Microphone for a Vocal Check In

Another interesting technique used frequently by G. and F. is a sort of grunt they emit orally while a student is speaking. The main objective seems to be to let students know they are being listened to and understood. In other words, this "vocal check-in" seems to satisfy a *phatic function*, the means by which two or more speakers reassure themselves that not only are they being listened to, but they are also being understood. The frequent vocal check-ins that occur online seem to substitute for other means through which the phatic function is carried out in physical classrooms.

OVERVIEW OF FINDINGS

The tools used in Meet were the chat, the microphone, the video camera, and, in and Jamboard, the laser highlight, the colored pens and the highlighters. Table 1 presents the main communication techniques, the visible parts of the schemes, the teachers used to communicate with these tools. I will now describe the main potentials and limitations that seemed to be experienced by the teachers and the students, and to mostly influence the emergence and elaboration of signs.

The chat offered the potential to "see behind" students' "raised hands". Indeed, in this online lesson, when students were asked a question, or when they wanted to say something in support or against a classmate's statement, they would signal their desire to intervene by writing in the chat. Scrolling down through the chat, the teachers could quickly get an idea of who was thinking what; perhaps this allowed them to identify relevant signs to pick out and share more rapidly than in the physical classroom. However, a limitation of having students quickly type their signs into the chat is that the signs emerged in a single modality (written verbal), and usually in a very concise and cryptic form.

The fact that the students could activate the microphone only without the video camera and without having direct access to the Jamboard was another limitation that forced students to share signs in a single modality (aural), when called on. However, this also led some students to attempt to clarify their signs and reasoning through a sort of aural enaction of pointing gestures, giving names to sets of marks on the board.

Writing or highlighting on the Jamboard has the potential of coordinating speech and gestures, like in a physical context, allowing the simultaneous production of sets of signs with graphical and aural components that are immediately accessible to the listeners. However, the fact that, as a class norm, the teacher did not allow students to act on the Jamboard caused the following asymmetry in the management of the written signs: the students could only see them, while the teachers (actually, only G.) could highlight and modify them. This asymmetry forced the students to verbally interpret the written signs, on the one hand, and, on the other hand, it led the teacher to translating them into a mixed verbal and graphical set of signs. The teachers were constantly active (more than in the physical classroom) interpreting the students' signs, guessing at the meanings evoked by the students, adding components and enriching the semiotic bundles, which led to a relatively large discrepancy between Al.'s intended meanings and those in the G.'s interpretation of the shared signs (Figure 4).

technique	*goal*
Open and scroll chat	Obtain an overview of students' participation and search for interesting signs for discussion
Write in chat	Parallel communication between students or between students and the teacher to organize speaking turns, or just to "stay in touch"
Write or highlight in Jamboard	Add and coordinate visual parts of signs in student's speech, to help class follow/focus and show speaker s/he is "understood"
Use mic. for vocal "check-in"	Let students know they are being listened to (phatic function)
Use mic. to call on "invisible students"	Increase participation of students (especially those expected to struggle)
Use mic. to aurally enact a pointing gesture	Get listeners' attention to go somewhere specific on screen

Table 1. Main communication techniques implemented in Jamboard and Google meet to communicate

In general, the analyses also suggested that focalization and provide a synthesis actions prevailed over the others. Frequently right after asking for a synthesis, online the teacher would quickly shift to providing a synthesis herself, while in her physical classroom, F. would carry out this action less frequently. This behavior could be due to a feeling of lack of control over her class that F. experienced online, or simply to the difficulty of bearing silence in the online situation.

DISCUSSION AND CONCLUSION

This study suggests that during the lockdown in Italy, even in a class whose teacher belongs to a research action group and who continued to collaborate with researchers (indeed one was even co-teaching with her!), digital technology is still used rather "naively", as a means to transfer online what the teacher usually implements in her physical classroom, without fully taking advantage of its potential. The analyses suggest that changes in how the discussion was conducted online as opposed to in physical classrooms were due to limitations in how signs were co-produced and shared between students and teachers, but probably also to the norms imposed during the crisis situation. Ruthven's decade-old claim seems to still apply:

> These developments have been readily embraced because they provide relatively simple (if often expensive) enhancements to everyday means of communication and resource use, in and beyond the classroom. These technologies are not strongly framed in didactic terms, and have potential to support activity across the didactic spectrum; nevertheless, in practice they are often appropriated to a reproductive didactic. (Ruthven, 2012, p. 629)

So, it seems natural to question the extent to which the online communication was really responsible for these changes? What could have been done to better exploit the potential of communication supported by digital technology and transform (at least some of) these constraints into opportunities?

A few possibly insightful directions to explore come to mind, concerning the chat and, more in general, shared spaces provided by digital technology. Chats could be used both synchronously and asynchronously with respect to the online discussion to foster students' written argumentations. Indeed, the chat tool offers a means of online communication that privileges written text. This tool could be used according to different schemes, based on forms of communication it is supposed to mediate (e.g., teacher to student, student to teacher; students to student, ...).

Shared "spaces" could be provided, such as text documents or boards shared within small groups of students, in order to promote the production of personal signs that can later be shared with the larger group. For example, breakout rooms as "secure" spaces where students can interact more freely without teacher mediation. In a shared space such as Jamboard, the management could be different, for example, allowing students to access it, write on it and produce online gestures around the signs on it, as was done only by G. in the online lesson. In contexts where the teacher prefers not to give students access to the board, such as F.'s, lessons could involve activities designed to foster the development of verbal language, for example asking for predictions and descriptions of the behavior of interactive digital artifacts (e.g., Baccaglini-Frank et al., 2018). This goes back to the first line of development of digital technologies described by Ruthven, concerning their mediational role with respect to specific mathematical content. While there has been lots of research in this direction over the past decades, why have classroom practices lagged so far behind, making teachers and students fall into such difficult situations (the one described is among the "happiest" I have witnessed during the lockdown) when online education became necessary?

A key might be to look in the direction of professional development. What kind of professional development is needed for teachers to be able to feel more comfortable in online teaching contexts? How should professional development courses be designed? These and similar questions have been raised in different countries and communities, who all feel the need for research on online teaching and learning with and through technology (Bakker et al., 2021). I join this chorus.

NOTES

1. At the Mathematics Department of the University of Pisa the research-action group "Gruppo di Ricerca e Sperimentazione in Didattica della Matematica" (GRSDM) directed by Anna Baccaglini-Frank and Pietro Di Martino has continued to meet online throughout the pandemic.

2. The conceptualization of algorithms as artifacts, and the didactic potential of their synergy are described in Baccaglini-Frank, et al. (2021).

3. This algorithm owes its Italian name to the collaboration between the research group coordinated by Paolo Boero and a Canadian group of teachers.

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REFERENCES

Antonini, S., Baccaglini-Frank, A., & Lisarelli, G. (2020). From experiences in a dynamic environment to written narratives on functions. *Digital Experiences in Mathematics Education, 6*(1), 1–29. https://doi.org/10.1007/s40751-019-00054-3

Arzarello, F. (2006). Semiosis as a multimodal process. *Revista Latinoamericana de Investigación en Matemática Educativa, 9*(1), 267–299.

Baccaglini-Frank, A. (2019). Dragging, instrumented abduction and evidence in processes of conjecture generation in a DGE. *ZDM Mathematics Education, 51*(5), 779–791. https://doi.org/10.1007/s11858-019-01046-8

Baccaglini-Frank, A. (2021). To tell a story, you need a protagonist: how dynamic interactive mediators can fulfil this role and foster explorative participation to mathematical discourse. *Educational Studies in Mathematics, 106*(2), 291–312. https://doi.org/10.1007/s10649-020-10009-w

Baccaglini-Frank, A., Di Martino, P., & Sinclair, N. (2018). Elementary school teachers' implementation of dynamic geometry using model lesson videos. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 99–106). PME.

Baccaglini-Frank, A., Funghi, S., Maracci, M., & Ramploud, A. (2021). "One times one, but actually they are ten times ten": Learning about multiplication and decimal notation in third grade by comparing algorithms. [Manuscript submitted for publication].

Baccaglini-Frank, A., & Mariotti, M.A. (2010). Generating conjectures in dynamic geometry: The maintaining dragging model. *International Journal of Computers for Mathematical Learning, 15*(3), 225–253. https://doi.org/10.1007/s10758-010-9169-3

Bakker, A., Cai, J., & Zenger, L. (2021). Future themes of mathematics education research: An international survey before and during the pandemic. *Educational Studies in Mathematics, 107*(1), 1–24. https://doi.org/10.1007/s10649-021-10049-w

Bartolini Bussi, M. G. (1998). Verbal interaction in mathematics classroom: A Vygotskian analysis. In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and Communication in Mathematics Classroom* (pp. 65–84). NCTM.

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom. In L. English, M. G. Bartolini Bussi, K. Jones, & R. Lesh, *Handbook of International Research in Mathematics Education* (2nd ed.) (pp. 746–783). Routledge/Taylor & Francis Group.

Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-geometry. *International Journal of Computers for Mathematical Learning, 6*(3), 283–317. https://doi.org/10.1023/A:1013309728825

Lisarelli, G., Baccaglini-Frank, A. & Di Martino, P. (2021). From how to why: A quest for the common mathematical meanings behind two different division algorithms. *Journal of Mathematical Behavior, 63*, 100897. https://doi.org/10.1016/j.jmathb.2021.100897

Mariotti, M. A. (2009). Artifacts and signs after a Vygotskian perspective: the role of the teacher. *ZDM Mathematics Education, 41*(4), 427–440. https://doi.org/10.1007/s11858-009-0199-z

Mariotti, M.A., & Maffia, A. (2018). From using artefacts to mathematical meanings: The teacher's role in the semiotic mediation process. *Didattica della Matematica. Dalla Ricerca alle Pratiche d'Aula, 3*, 50–64.

Mariotti M.A., & Maracci, M. (2010). Un artefact comme outils de médiation sémiotique: une ressource pour l'enseignant. In G. Gueudet & L. Trouche (Eds.), *Ressources Vives. Le Travail Documentaire des Professeurs en Mathématiques* (pp. 91–107). Presses Universitaires de Rennes; INRP.

Michaels, S., & O'Connor, C. (2015). Conceptualizing talk moves as tools: Professional development approaches for academically productive discussion. In L. B. Resnick, C. S. C. Asterhan, & S. N. Clarke (Eds.), *Socializing Intelligence through Talk and Dialogue* (pp. 347–362). AERA. https://doi.org/10.3102/978-0-935302-43-1_27

Monaghan, J., Trouche, L., & Borwein, J. M. (2016). *Tools and mathematics: Instruments for learning*. Springer. https://doi.org/10.1007/978-3-319-02396-0

Ramploud, A., Funghi, S., & Mellone, M. (2021). The time is out of joint. Teacher subjectivity during COVID-19. *Journal of Mathematics Teacher Education*. https://doi.org/10.1007/s10857-021-09506-3

Ruthven, K. (2012). The didactical tetrahedron as a heuristic for analysing the incorporation of digital technologies into classroom practice in support of investigative approaches to teaching mathematics. *ZDM Mathematics Education*, *44*(5), 627–640. https://doi.org/10.1007/s11858-011-0376-8

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Yerushalmi, H. (2007). Paradox and personal growth during crisis. *The American Journal of Psychoanalysis, 67*(4), 359–380. https://doi.org/10.1057/palgrave.ajp.3350038