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Threshold anomalies in Horava–Lifshitz-type theories

Giovanni Amelino-Camelia *, Leonardo Gualtieri, Flavio Mercati

Dipartimento di Fisica, Università di Roma "La Sapienza" and Sez. Roma 1 INFN, P.le A. Moro 2, 00185 Roma, Italy

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ABSTRACT

Recently the study of threshold kinematic requirements for particle-production processes has played a very significant role in the phenomenology of theories with departures from Poincaré symmetry. We here specialize these threshold studies to the case of a class of violations of Poincaré symmetry which has been much discussed in the literature on Horava–Lifshitz scenarios. These involve modifications of the energy–momentum ("dispersion") relation that may be different for different types of particles, but always involve even powers of energy–momentum in the correction terms. We establish the requirements for compatibility with the observed cosmic-ray spectrum, which is sensitive to the photopion-production threshold. We find that the implications for the electron–positron pair-production threshold are rather intriguing, in light of some recent studies of TeV emissions by Blazars. Our findings should also provide additional motivation for examining the fate of the law of energy–momentum conservation in Horava–Lifshitz-type theories.

1. Introduction

Among the first results produced by the recent effort of investigation of the Horava–Lifshitz scenario (see, e.g., Refs. [1–10]) of particular interest from the phenomenology perspective are those that concern possible violations of Poincaré symmetries. In fact, over the last decade our ability to test Poincaré symmetry (particularly, but not exclusively, its Lorentz sector) has improved very significantly, and we are now in a position to investigate even very tiny violation effects (see, e.g., Refs. [11–23]).

While a detailed understanding of the nature of the Poincaré-symmetry violations in Horava–Lifshitz-type scenarios has still not been reached, mostly because of challenges from a renormalization-group perspective [4,5], some consensus appears to be emerging at least on some features that can be used, as already stressed by other authors [6–10], for a preliminary phenomenological analysis. The feature on which we shall focus is the presence of modifications of the energy/momentum (dispersion) relation of the type

$$E^2 = m_i^2 + p^2 + \lambda_i^{(2)} p^4 + \lambda_i^{(4)} p^6,$$

where the dimensionful parameters $\lambda_i^{(n)}$ carry an index i which denotes a possible "non-universality" of the effects (effects that have different magnitude for different particles) and an index (n) which simply refers to the number of length dimensions (e.g., $\dim[\lambda_i^{(2)}] = [l^2]$).

The fact that this type of dispersion relations may emerge in the Horava–Lifshitz scenario has been argued by several authors (see, e.g., Refs. [6–10]). We here intend to expose some opportunities that are provided by the presence of this type of modification of dispersion relations, particularly for what concerns the study of "threshold anomalies" [18], i.e. the study of the implications of violations of Poincaré symmetry for the kinematic conditions at the threshold for some particle-creation interactions.

The study of threshold anomalies as a possible signal of Poincaré-symmetry violation has already been explored within other quantum-gravity inspired formalisms (see, e.g., Refs. [13,15–19]). But while these previous studies of threshold anomalies focused on the possibility that dimensionful parameters such as $\lambda_i^{(2)}$ and $\lambda_i^{(4)}$ be set by the Planck scale (so that, e.g., $\lambda_i^{(2)} \sim 1/E_{\rm Planck}^2 \simeq 10^{-56}\ {\rm eV}^{-2}$), the Horava–Lifshitz scenario, in which $\lambda_i^{(2)}$, $\lambda_i^{(4)}$ are not directly linked to the Planck scale, provides motivation for a phenomenology of broader scopes.

From the Horava–Lifshitz perspective, another limitation of previous phenomenological analyses of threshold anomalies concerns the handling of the energy–momentum conservation law. In other frameworks with violations of Poincaré symmetry only the case of unmodified energy–momentum conservation was considered. Modifications of energy–momentum conservation were considered in several studies but only when attempting to ultimately restore (at least deformed) relativistic invariance, in the sense of the "Doubly Special Relativity" proposal [24–26]. Also concerning the possibility of violations of energy–momentum conservation the present understanding of the Horava–Lifshitz framework exposes the need for more general analyses, since it involves modified dispersion re-

* Corresponding author.
E-mail address: amelino@roma1.infn.it (G. Amelino-Camelia).

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lations within a framework that also appears to involve violations of translational symmetries [3,27], which may well produce violations of energy–momentum conservation.

In the next section we consider threshold anomalies for the process of electron–positron pair production in photon–photon collisions, and find that our Horava–Lifshitz-inspired analysis leads to a picture that could produce an increase in our expectations for the spectrum of multi-TeV photons to be observed from certain Blazars. Since it has been argued [17,28] that indeed the abundance of multi-TeV photons observed from certain Blazars is unexpectedly high this may be a valuable opportunity for Horava–Lifshitz phenomenology. Section 3 discusses an analogous threshold anomaly for photopion production, which is relevant for the observations of ultra-high-energy cosmic rays, and provides the basis for additional insight on what type of Horava–Lifshitz-inspired models could provide the most fruitful phenomenology, while preserving consistence with available experimental data. While Sections 2 and 3 take as working assumption the fact that possible effects of modification of the law of energy–momentum conservation can be neglected at the level of our leading-order analysis, in Section 4 we discuss the differences in the description of threshold anomalies that would instead arise if leading-order effects of modification of energy–momentum conservation were to be found. The main objective of Section 4 is therefore the one of highlighting the significance for phenomenology of the analysis of violations of translational symmetry in the Horava–Lifshitz scenario, which unfortunately has so far not attracted much attention. Section 5 offers some closing remarks.

2. Pair-production threshold anomalies

The study of the threshold kinematic requirements for the pair-production process, $\gamma\gamma \to e^+e^-$, has important implications for the opacity of the Universe to photons, which in turn can be indirectly studied observationally. In previous quantum-gravity-motivated studies [16–19] of anomalies for the pair-production threshold it was already observed that violations of Poincaré symmetry can be particularly significant for the study of absorption of multi-TeV photons (photons with energies between a few and, say, 30 TeV) by the infrared diffuse extragalactic background. In this section we intend to specialize this observation to the case of the Horava–Lifshitz-inspired phenomenological framework described in our introductory remarks, centered on a modification of the dispersion relation.

The fact that we plan to obtain results relevant for collisions between a multi-TeV photon and a photon in the infrared diffuse extragalactic background invites us to consider the case of a collision in which one of the photons is hard, with energy–momentum E, P such that $E \gg m_e$ (denoting with m_e the electron mass), whereas the other photon is soft, with energy–momentum ϵ, p such that $\epsilon \ll m_e$. Of course, for fixed value of the soft-photon energy ϵ (representative of photons in the infrared diffuse background) the production of an electron–positron pair is possible only for values of the hard-photon energy E greater than a certain minimum (threshold) value, which we can denote with E^*_ϵ. Within ordinary Poincaré covariant kinematics one easily finds that the threshold requirement is $E > E^*_\epsilon = m_e^2/\epsilon$, but it is known [13,15–19] that this result can be affected rather sizably even by small departures from Poincaré symmetry.

In order to establish the size of the threshold anomaly for the case of our Horava–Lifshitz-inspired framework we shall of course make use of the modified dispersion relation already discussed in the introductory remarks. For the hard photon we therefore have

$$E \simeq P + \frac{1}{2}\lambda_\gamma^{(n)} P^{n+1}, \tag{1}$$

where n can be 2 or 4. We are only aiming for a description of the dominant correction to the threshold requirement, so we will only consider $n = 2$ whenever its implications are not negligible with respect to the ones of the $n = 4$ terms, and in turn consider exclusively the $n = 4$ terms if the contributions from terms with $n = 2$ can be neglected. We assume of course that the $\lambda_i^{(n)}$ are all small, and in particular in the analysis of the pair-production threshold we assume $\lambda_{\gamma,e}^{(2)} \ll (100\ \text{TeV})^{-2}$ and $\lambda_{\gamma,e}^{(4)} \ll (100\ \text{TeV})^{-4}$. On dimensional ground one might guess $\lambda_i^{(4)} \sim (\lambda_i^{(2)})^2$, and whenever $\lambda_i^{(4)} \lesssim (\lambda_i^{(2)})^2$ the effects of the $\lambda_i^{(4)}$ parameters can be neglected at leading order. As already observed in Ref. [6], only in the case $\lambda_i^{(4)} \gg (\lambda_i^{(2)})^2$ the $\lambda_i^{(4)}$ parameters produce the dominant effects.

Consistently with the scopes of our leading-order analysis we can neglect the modification of the soft-photon dispersion relation,

$$\epsilon \simeq p + \frac{1}{2}\lambda_\gamma^{(n)} p^{n+1} \simeq p, \tag{2}$$

since we are interested in the case of $p \ll P$, which of course implies that $\lambda_\gamma^{(n)} p^{n+1} \ll \lambda_\gamma^{(n)} P^{n+1}$.

For the outgoing electron (positron) we introduce the notation E_- (E_+) for its energy and p_- (p_+) for its spatial momentum, so that

$$E_\pm \simeq p_\pm + \frac{m_e^2}{2p_\pm} + \frac{1}{2}\lambda_e^{(n)} p_\pm^{n+1}, \tag{3}$$

where we also used the fact that the electron–positron pairs produced at threshold in collisions between a multi-TeV photon and a photon in the infrared diffuse extragalactic background are inevitably ultra-relativistic ($p_\pm \gg m_e$).

The kinematic requirements at threshold are the ones that require the minimum energies for the process to occur and as a result the process at threshold inevitably is a head-on collision [18] (collisions that are not head-on always "cost" more energy, which "pays for" the additional components of momentum). This simplifies the analysis since for the purpose of establishing the threshold requirements one can exploit the fact that the whole process occurs along one spatial direction. We can therefore efficaciously reason in terms of the modulus of the spatial momenta, and write energy–momentum conservation as follows:

$$\begin{cases} E + \epsilon = E_+ + E_-, \\ P - p = p_+ + p_-. \end{cases} \tag{4}$$

Using the dispersion relations (1) and (3) in the equation of conservation of energy one finds

$$P + \frac{1}{2}\lambda_\gamma^{(n)} P^{n+1} + \epsilon = p_+ + p_- + \frac{m_e^2}{2p_+} + \frac{m_e^2}{2p_-} + \frac{1}{2}\lambda_e^{(n)}\left(p_+^{n+1} + p_-^{n+1}\right), \tag{5}$$

which can then be cast in the form

$$\frac{1}{2}\lambda_\gamma^{(n)} P^{n+1} + 2\epsilon = \frac{m_e^2}{2p_+} + \frac{m_e^2}{2p_-} + \frac{1}{2}\lambda_e^{(n)}\left(p_+^{n+1} + p_-^{n+1}\right) \tag{6}$$

using the equation of conservation of spatial momentum and the fact that $\epsilon \simeq p$.

Next we observe that at zero-th order in $\lambda_\gamma^{(n)}$, $\lambda_e^{(n)}$ (i.e. in the standard Poincaré covariant derivation) one obtains from these equations $p_+ = p_- \simeq P/2$. This can be exploited in our first-order derivation by allowing us to observe that

$$\frac{m_e^2}{p_+} + \frac{m_e^2}{p_-} \simeq \frac{4m_e^2}{P}, \qquad \lambda_e^{(n)} p_\pm^{n+1} = \lambda_e^{(n)}\left(\frac{P}{2}\right)^{n+1}, \tag{7}$$

neglecting terms on magnitude not greater than $\lambda^{(n)}P^{n+1}m_e^2/P^2$ which of course can be neglected in our derivation focusing on the dominant $\lambda^{(n)}P^{n+1}$ corrections (m_e/P is indeed very small for the collisions we intend to investigate). Using (7) one obtains from (6) the following result

$$P+\lambda_\gamma^{(n)}\frac{P^{n+2}}{4\epsilon}\simeq\frac{m_e^2}{\epsilon}+\lambda_e^{(n)}\frac{P^{n+2}}{2^{n+2}\epsilon}. \tag{8}$$

And in turn this allows us to conclude that, for fixed soft-photon energy ϵ, the pair-production process is possible within our Horava–Lifshitz-inspired framework only when $E>E_\epsilon^*$, where the threshold energy E_ϵ^* is solution of the equation

$$E_\epsilon^*+\left(\lambda_\gamma^{(n)}-\frac{\lambda_e^{(n)}}{2^n}\right)\frac{(E_\epsilon^*)^{n+2}}{4\epsilon}\simeq\frac{m_e^2}{\epsilon}. \tag{9}$$

The standard Poincaré covariant result $E_\epsilon^*=m_e^2/\epsilon$ is of course recovered in the limit $\lambda_\gamma^{(n)},\lambda_e^{(n)}\to 0$. For $\lambda_\gamma^{(n)}>\lambda_e^{(n)}/2^n$ one finds lower values of E_ϵ^*, while for $\lambda_\gamma^{(n)}<\lambda_e^{(n)}/2^n$ one obtains values of E_ϵ^* that are greater than m_e^2/ϵ.

Analogous relations among parameters of schemes with particle-dependent (non-universal) modifications of the dispersion relations have already been derived (see, e.g., Ref. [19]), but typically assuming symmetry-breaking scales of the order of the Planck scale ($\sim 10^{28}$ eV) and different dependence on energy. Taking $E\sim 10$ TeV and $\epsilon\sim 0.04$ eV one easily verifies that, in order for our Horava–Lifshitz-inspired framework to have observably large implications in this pair-production analysis, the scales $\lambda_\gamma^{(n)}$ and/or $\lambda_e^{(n)}$ should not be set by the Planck scale but by a much lower scale. For example in the case $n=2$ one would need $|\lambda_{\gamma,e}^{(2)}|\gtrsim(10^{20}\text{ eV})^{-2}$.

Preliminary indications on whether values higher or lower than m_e^2/ϵ could be favored experimentally can be obtained using data on the opacity of the Universe for multi-TeV photons. A high energy photon propagating in the intergalactic space can indeed interact with photons in the infrared diffuse extragalactic background, producing an electron–positron pair. The mean free path of 10 TeV photons depends on the spectrum of the infrared background photons in the range from $\simeq 0.03$ eV to $\simeq 0.08$ eV, with particularly strong dependence on the spectrum around 0.04 eV. And these estimates scale linearly with the (inverse of) the energy of the incoming hard photon. Unfortunately, it is difficult to determine the infrared diffuse extragalactic background, since direct measurements are problematic, owing to the presence of the bright Galactic and Solar System foregrounds [28]. Still it is noteworthy that in recent years there have been several reports (see, e.g., Refs. [28–30] and references therein) of spectra of some observed Blazars that appear to be harder than anticipated on the basis of the expected infrared-background absorption. One could therefore tentatively argue that the case of values of the pair-production threshold that are somewhat higher than m_e^2/ϵ, i.e. the case $\lambda_\gamma^{(n)}\lesssim\lambda_e^{(n)}/2^n$, finds some encouragement in the, however preliminary, observational situation. But this possibility must be contemplated very cautiously since the presence of anomalies is in no way necessary [31]. The observational situation does establish more robustly that values of the pair-production threshold lower than m_e^2/ϵ are objectively disfavored [18], so that Horava–Lifshitz scenarios with $\lambda_\gamma^{(n)}>\lambda_e^{(n)}/2^n$ (and $|\lambda_{\gamma,e}^{(2)}|\gtrsim(10^{20}\text{ eV})^{-2}$) appear to be excluded.

3. Photopion-production threshold anomalies

In the preceding subsection we discussed the implications of Horava–Lifshitz deformed dispersion relations for the process $\gamma\gamma\to e^+e^-$, but of course this is not the only process in which deformations to dispersion relations can produce significant threshold anomalies. In particular, there has been strong interest [13,16–19] in the analysis of the threshold requirements for the "photopion-production" process, $p\gamma\to p\pi$, and their relevance for the observed high-energy portion of the cosmic-ray spectrum.

The analysis of the photopion-production threshold is of course completely analogous to the one of the pair-production threshold, but it is slightly more tedious: in the case of $\gamma\gamma\to e^+e^-$ the calculations are simplified by the fact that both outgoing particles have the same mass and both incoming particles are massless, whereas for the threshold conditions for the photopion-production process one needs to handle the kinematics for a head-on collision between a soft photon of energy ϵ and a high-energy particle of mass m_p and momentum P_p producing two (outgoing) particles with masses m_p, m_π and momenta P'_p, P_π. Since however these additional complications pose no conceptual and no significant technical challenges (and a dedicated derivation of the photopion-production threshold with Poincaré-symmetry violations is given in Ref. [18]) we shall here just note the final result for the threshold condition in our Horava–Lifshitz-inspired framework:

$$\begin{aligned}E_\epsilon^*&+\frac{(E_\epsilon^*)^{2+n}}{4\epsilon}\\&\times\left[\lambda_p^{(n)}-\lambda_p^{(n)}\left(\frac{m_p}{m_p+m_\pi}\right)^{n+1}-\lambda_\pi^{(n)}\left(\frac{m_\pi}{m_p+m_\pi}\right)^{n+1}\right]\\&\simeq\frac{(m_p+m_\pi)^2-m_p^2}{4\epsilon}\end{aligned} \tag{10}$$

(neglecting of course all terms suppressed by both the smallness of $\lambda_i^{(n)}$ and the smallness of ϵ and/or $m_{p,\pi}$).

Introducing the notation $\mu_p\equiv m_p/(m_p+m_\pi)\simeq 0.9$ and $\mu_\pi\equiv m_\pi/(m_p+m_\pi)\simeq 0.1$ one therefore concludes that, for fixed soft-photon energy ϵ, when $\lambda_p^{(n)}(1-\mu_p^{n+1})>\lambda_\pi^{(n)}\mu_\pi^{n+1}$ the energy of the incoming proton required at threshold for photopion production is shifted toward lower values (in comparison to the standard case $\lambda_p^{(n)}=\lambda_\pi^{(n)}=0$), whereas when $\lambda_p^{(n)}(1-\mu_p^{n+1})<\lambda_\pi^{(n)}\mu_\pi^{n+1}$ this threshold energy is shifted toward higher values.

An exciting aspect of these threshold analyses for photopion production and the cosmic-ray spectrum is that they in principle provide access to scales of violation of Poincaré symmetry that are extremely high. For example, from (10) it is easy to infer that detailed studies of the cosmic-ray spectrum at energies $\gtrsim 10^{19}$ eV could allow us to probe values of $\lambda_p^{(2)}$ and $\lambda_\pi^{(2)}$ such that $|\lambda_{p,\pi}^{(2)}|\gtrsim(10^{30}\text{ eV})^{-2}$.

The feature of the cosmic-ray spectrum that can be most valuable from this perspective is associated with the Greisen–Zatsepin–Kuzmin (GZK) cutoff, which is essentially obtained as the threshold energy ($\sim 5\cdot 10^{19}$ eV) for cosmic-ray protons to produce pions in collisions with CMBR photons. The observational determination of the cosmic-ray spectrum has recently improved rather significantly as a result of observations conducted with the Pierre Auger cosmic-ray observatory [32]. There is no evidence of any shift of the GZK threshold within the accuracy so far achieved in determining the cosmic-ray spectrum, but the most promising outlook from the perspective of possible Poincaré violations is the one discussed in Ref. [33], which, within the framework here considered, would require $\lambda_p^{(n)}(1-\mu_p^{n+1})<\lambda_\pi^{(n)}\mu_\pi^{n+1}$. This scenario of Ref. [33] ensures consistency with available cosmic-ray-spectrum data and predicts a sort of "recovery" [33] of the spectrum at energies not much higher than the GZK scale. The prospects are therefore rather intriguing since a better determination of the beyond-GZK portion of the spectrum appears to be within the reach of studies planned for the Pierre Auger observatory.

4. A possible role for modifications of energy–momentum conservation

The results we derived so far assume that in the Horava–Lifshitz scenario there are no modifications of the law of energy–momentum conservation that could be large enough to affect the threshold requirements at the leading $\lambda^{(n)} P^{n+1}$ order. Previous studies [13,15–19] of the phenomenology of threshold anomalies due to violations of Poincaré symmetry mainly focused on the possibility of new physics affecting exclusively the Lorentz sector, so that the law of energy–momentum conservation would be unaffected. However, in the Horava–Lifshitz scenario the four-dimensional diffeomorphism invariance is broken down to foliation-preserving diffeomorphisms

$$\delta x^i = \zeta^i(t, \mathbf{x}), \qquad \delta t = f(t), \tag{11}$$

a subgroup which preserves the foliation structure of space-like slices. Therefore, as pointed out in several studies (see, e.g., Refs. [3,27]), local energy–momentum conservation is restricted to the spatial components. In a locally inertial frame, the theory is invariant under space translations but not under time translations, so that in principle energy might not be conserved.

Presently the literature still does not provide any guidance on the magnitude of the violations (if any) of energy conservation in particle-physics processes within the Horava–Lifshitz framework. But it is important for us to stress that our results could be significantly changed if these violations happen to be relevant, also hoping that this observation might motivate a more intense phase of study by the community of the issue of energy conservation in the Horava–Lifshitz framework.

For our exclusively illustrative purposes here it is sufficient to make a simple ansatz for a modified law of energy–momentum conservation, applicable to the case of electron–positron pair production in collisions between two photons:

$$\begin{cases} E + \epsilon - \Delta^{(2)}(E\epsilon^2 + E^2\epsilon) = E_+ + E_- - \Delta^{(2)}(E_+^2 E_- + E_+ E_-^2), \\ P - p = p_+ + p_- \end{cases} \tag{12}$$

where $\Delta^{(2)}$ is a parameter with length-squared dimensions. We use this recipe to obtain a rough estimate of the size of the threshold-anomaly effects that could be induced by violations of energy conservation with P^3 behaviour. And we shall be satisfied showing the implications of the parameter $\Delta^{(2)}$ for the case of the pair-production threshold, focusing on the dispersion-relation parameters with $n = 2$, $\lambda_\gamma^{(2)}$ and $\lambda_e^{(2)}$. Adopting the $\Delta^{(2)}$-deformed energy–momentum conservation, the derivation of the pair-production threshold requirement (which of course once again follows exactly the same steps described in Section 2) leads to the result

$$E_\epsilon^* + \left(\frac{\Delta^{(2)}}{2} + \lambda_\gamma^{(2)} - \frac{\lambda_e^{(2)}}{4}\right)\frac{(E_\epsilon^*)^4}{4\epsilon} \simeq \frac{m_e^2}{\epsilon}. \tag{13}$$

This shows that modifications of the law of energy–momentum conservation of magnitude comparable to the one we illustratively considered, and parametrized with $\Delta^{(2)}$, could affect the result for the threshold at the same level as the $\lambda_i^{(2)}$ parameters of modification of the dispersion relation. In principle one could even have cases in which the modification of the dispersion relation and the modification of the law of energy–momentum conservation balance each other ($\Delta^{(2)} = \lambda_e^{(2)}/2 - 2\lambda_\gamma^{(2)}$) giving the net result of no leading-order correction to the threshold requirements. Such a cancellation is actually expected [24,34] in frameworks based on the concept of "Doubly Special Relativity" [24–26], where one could accommodate modifications of the dispersion relation within a model which is still fully relativistic, but relativistic in a deformed sense (with two non-trivial relativistic invariants, a speed scale and a length scale, rather than one). But such a cancellation is not to be expected [34] in frameworks in which instead Poincaré symmetry is genuinely broken (rather than deformed) as appears to be the case of the Horava–Lifshitz framework. So, while we cannot exclude that investigations of the fate of the relevant diffeomorphism-invariance issues may lead to a reassessment of the quantitative aspects (magnitude) of the threshold anomalies we here considered, we do expect these threshold anomalies to be a genuine characteristic of the Horava–Lifshitz framework.

5. Closing remarks

In spite of a vigorous effort, composed of a large number of dedicated studies in just a short time, the understanding of the physics of the Horava–Lifshitz scenario appears to be still far from taking final shape. There is however growing consensus on some aspects, and particularly on the presence of modifications of the dispersion relation of the type we here studied. The threshold anomalies we analyzed represent challenges and opportunities which may provide guidance, and perhaps even encouragement, for further studies of the framework.

From a phenomenology perspective interest in this scenario can originate from the rather natural emergence of "non-universal effects" (different magnitude for different type of particles), but in ways that one can imagine to become predictive at a later more mature stage of investigation. Particularly interesting from our perspective is the possibility that one might find that the implications of the Horava–Lifshitz scenario are different for particles of different spin, since our analysis involved particles with spin 1, 1/2 and 0 (i.e. γ, $e^\pm$, p, π). For example, the most intriguing aspect of our analysis concerns the pair-production threshold, where the observations appear to invite (however prudently) consideration of the possibility of new fundamental physics. The requirement we obtained, $\lambda_\gamma^{(n)} \lesssim \lambda_e^{(n)}/2^n$, would carry little significance if one ended up introducing it by hand in the Horava–Lifshitz scenario, since it would then amount to a standard observationally-imposed constraint on a potentially rather large parameter space. But the present limited understanding of the framework, particularly for what concerns issues connected with the renormalization group [4], appears to leave open the possibility that such a condition be derived as an inevitable feature of the Horava–Lifshitz setup. In that case the evaluation of compellingness of the proposal should clearly take into account the type of phenomenological implications that we here focused on.

Of similar nature is our contribution on the points concerning the law of energy–momentum conservation. In that respect the most interesting aspect from the phenomenology perspective originates from the fact that the Horava–Lifshitz scenario might host both modifications of the dispersion relation, of a type that is not too different from the ones already considered in other Poincaré-violation scenarios, and modifications of the law of energy–momentum conservation, which is instead a possibility that had been mostly neglected in previous studies of Poincaré-violation scenarios. We observed here that there could be a strong dependence of a meaningful observable aspect (our threshold anomalies) on possible violations of the law of energy–momentum conservation, also hoping to provide motivation for an increased effort of investigation of the fate of translational symmetries in the Horava–Lifshitz scenario. In spite of the large number of studies devoted to this proposal, only very few authors appear to have considered the implications for translational symmetries, which instead, in ways that our analysis renders more tangible, will probably play a key role in assessing the compellingness of the physical picture produced by the Horava–Lifshitz scenario.

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