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A comparative study of wear laws for soft-on-hard hip implants using a mathematical wear model

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Abstract

Wear of UHMWPE acetabular components is the most serious threat to the long-term success of hip replacements. Consequently numerical and experimental wear simulations are of great interest. The present study proposes a mathematical wear model and compares the most recent wear laws, based on the so-called cross-shear (CS) effect. Simulations highlighted the variability of wear predictions with the wear factors/laws. Moreover a sensitivity analysis underlined that the kinematic conditions affect volumetric/linear wear more than the loading ones. This study confirms the importance of the CS in wear predictions even if some critical issues are still open, requiring further investigations.

Keywords: hip replacement, wear prediction, cross-shear, wear model

1. Introduction

Wear of conventional Ultra High Molecular Weight Polyethylene (UHMWPE) has historically been recognized as the primary cause of soft-on-hard (SoH) hip implant failure and revision. Attested studies have demonstrated that around 100 millions of submicron UHMWPE particles are released into the periprosthetic fluid for a daily patient activity [1]. This wear debris typically triggers a cascade of adverse tissue reactions leading to osteolysis and implant loosening [2].

In the last decades this limitation has been motivating researchers in investigating wear mechanisms of UHMWPE, mainly by means of experimental tests. Among these, clinical and laboratory studies can be distinguished. The former are based on in-vivo estimation of linear wear by radiography [3, 4] or stereo-photogrammetry [5], or on in-vitro wear assessment of retrieved components [6–10]. As far as laboratory studies is concerned, hip joint simulators are being continuously improved for replicating realistic loading and kinematic conditions, e.g. [11, 12]. Pre-clinical experimental wear predictions are certainly a useful and necessary step in the optimization of the design process of Total Hip Replacement (THR), but they can be highly time/cost demanding and allow mainly short-term wear predictions.

In order to bypass these drawbacks, theoretical and numerical approaches can be adopted. Several computational wear models of THRs can be found in the literature, as reviewed in [13], which allow long term wear predictions. The first successful wear simulation of MoP implants was pioneered by Maxian et al. in 1996 [14] and was based on the Archard wear law. Since then, numerical simulations have been carried out to investigate the influence on wear of some design factors such as head size and cup thickness [15, 16], metal backing [17], surface roughness [16, 18], friction coefficient [18] and body weight [16].

Recent understandings of the wear mechanisms of UHMWPE made a break-through towards an improved THR wear modelling possible. It was demonstrated by Wang et al. [19] and by Turell et al. [20] that the UHMWPE-metal wear is strongly affected by multi-directional sliding, known as the Cross-Shear (CS) effect. In addition, other studies, such as [21, 22], showed the effect of the contact pressure on the wear factor. More recently, these findings were introduced in the Archard wear law by defining the wear factor as a function of the so-called cross-shear ratio ($CS_r$) or both of the CS, and the local contact pressure, as reported by Kang et al. in [23] and [24], respectively. In addition, experimental works carried out by Wang [25] showed the elastic nature of the deformation of UHMWPE micro-asperities under contact, yielding to a weak validity of the Archard wear law for MoP implants. Consequently a new wear law was presented by Liu et al. [26] and applied to MoP hip replacements, thus obtaining wear predictions in improved agreement with experimental data.

It is worth noting that most of the wear simulations mentioned above, were carried out using commercial Finite Element (FE) codes, integrated with user-defined subroutines by means of a procedure that can be quite complex and computationally expensive. Only rarely an analytical approach was pursued, such as by Raimondi et al. [16], whose model for MoP implants was based on Hertzian pressure distribution and a simplified expression of the sliding distance.

In the current study a mathematical parametric model for SoH hip implants is presented, improved with respect to the one described in [16] in several aspects, such as the contact analysis...
and the sliding distance evaluation. The main aim of the paper is to compare, by means of the developed model, the most recent wear laws for UHMWPE in terms of volumetric/linear wear predictions. Indeed such wear laws were generally implemented in models with different characteristics (e.g. geometries, material properties or kinematic/loading conditions), so their results are not easily comparable. The sensitivity of such wear laws to different kinematic and loading conditions is also discussed.

2. Wear laws for soft-on-hard hip implants

Wear modelling of SoH implants still remains an open issue and needs both the assumption of simplifying hypotheses and experimental findings.

Generally it is assumed that the wear is localized in the plastic cup, softer than the metallic head. Moreover the surfaces are considered in dry contact, the lubrication effect being taken into account in the wear factor empirically derived. In addition, although several wear mechanisms can affect the implant during in-vivo functioning (e.g. fatigue, corrosive wear), only adhesion and abrasion are considered [27]. Under this last hypothesis the Archard wear law holds, stating that, for a transversal load cycle, there is a proportionality, through a wear factor hypothesis the Archard wear law holds, stating that, for a transversal load cycle, there is a proportionality, through a wear factor $k_f$, between the volumetric wear $V$ and the product of the resultant normal load $L_N$ and the sliding distance $s$

$$V = k_f L_N s$$  \hspace{1cm} (1)

Often the Archard wear law is more conveniently written in a local instantaneous form yielding the linear wear rate $h_i(P, t)$ at a point $P$ of the cup surface at a time $t$

$$h_i(P, t) = k_f(P, t) p(P, t) |v(P, t)|$$  \hspace{1cm} (2)

where $p(P, t)$ is the instantaneous contact pressure and $v(P, t)$ the sliding velocity.

The wear factor is evaluated experimentally by means of pin-on-disc/plate or hip joints simulator wear tests and depends on several factors such as coupled materials, surface roughness, lubrication conditions, hence on the experimental set-up as well. A constant wear factor was used in the first wear simulations by Maxian et al. [14, 15], based on data from [28]. Raimondi et al. [16] considered $k_f$ function of the head roughness $R_n$, according to results reported in [29].

In the last fifteen years, experimental observations on the UHMWPE wear, e.g. [19], stressed that when polyethylene is subjected to multi-directional sliding against a metallic counterface, the polymeric chains acquire a principal molecular orientation (PMO) in whose direction the surface results harder and the wear resistance increases. On the opposite, in the direction perpendicular to the PMO, i.e. the CS direction, there is a strain softening phenomenon which is predominantly responsible for the detachment of fibrous wear debris from the worn surfaces. The concept is schematically represented in Fig. 1.

In this paper the cross-shear ratio $CS_r(P)$ is defined, in agreement with other studies in the literature [30, 23, 24, 26], as the ratio between the frictional work released perpendicularly to the PMO ($W_{\perp}$) and the total frictional work ($W_{tot}$) in a load cycle of time period $T$, that is

$$CS_r(P) = W_{\perp}(P)/W_{tot}(P)$$  \hspace{1cm} (3)

with

$$W_{\perp}(P) = \int_0^T f(P, t)p(P, t) v(P, t)|v(P, t)| \sin^2 \zeta(P, t)dt$$  \hspace{1cm} (4)

$$W_{tot}(P) = \int_0^T f(P, t)p(P, t) v(P, t)dt$$  \hspace{1cm} (5)

Such a definition gives the so called cycle-averaged CS. In the above equations $f(P, t)$ is the friction coefficient, which can vary in place and in time, and $\zeta(P, t)$ is the angle between $v(P, t)$ and the PMO (of unit vector $\mu$), that is

$$\cos \zeta(P, t) = \frac{\mu(P) \cdot v(P, t)}{|v(P, t)|}$$  \hspace{1cm} (6)

However, the PMO direction and therefore $\zeta(P, t)$ are not known a priori, but they require the solution of an optimization problem, being $\mu(P)$ the direction which minimizes $CS_r(P)$ or, alternatively, $W_{\perp}(P)$.

As it is commonly assumed that $f(P, t)$ does not vary within a cycle, i.e. $f(P, t) = f(P)$, Eq. 3 can be simplified yielding a constant friction cross-shear ratio $CS_f$:

$$CS_f(P) = \frac{\int_0^T f(P) p(P, t) |v(P, t)| \sin^2 \zeta(P, t)dt}{\int_0^T f(P) p(P, t) |v(P, t)| dt}$$  \hspace{1cm} (7)

Furthermore in the literature, e.g. [30, 23], another simplifying hypothesis is commonly adopted, that is the replacement of the instantaneous pressure with its average value $\bar{p}(P)$ over a load cycle. This leads to the following version of the CS ratio, denoted $CS_s$, commonly employed in the literature

$$CS_s(P) = \frac{\int_0^T |v(P, t)| \sin^2 \zeta(P, t)dt}{\int_0^T |v(P, t)| dt}$$  \hspace{1cm} (8)

It is worth noting that $CS_s$ depends only on the kinematic conditions and ranges between 0 and 0.5, the former corresponding to a unidirectional sliding while the latter to circular/squared tracks or whenever $W_{\perp} = W_{\parallel}$, $W_{\parallel}$ is work released in the direction parallel to the PMO; it is also $W_{tot} = W_{\perp} + W_{\parallel}$.
Such a quantification of the CS effect enabled Kang et al. [23] to introduce a more complex expression of the wear factor

\[ k_f(CS_s) = 3.28 \times 10^{-7}\ln(CS_s) + 1.62 \times 10^{-6} \]  

As in the above equation, in the following the dependence on the point \( P \) will be omitted for brevity, but it should be reminded that the wear factor is function of the cup point through the CS ratios, i.e. \( k(CS_s, P) \).

Barbour et al. [21] and Wang et al. [22] evidenced that the wear factor of SoH hip implants depends also on the contact pressure. Hence, in 2009, Kang and her research group revised Eq. 9 and proposed a wear factor function of both \( CS_s \) and \( \bar{p} \) [24]

\[ k_f(CS_s, \bar{p}) = \exp[13.1 + 0.9\ln(CS_s) - 0.29 \bar{p}] \]  

Experimental tests showed that, rather counter intuitively, the wear rate decreases as the average contact pressure increases. This can be explained considering the nature of deformation of the plastic cup. Indeed Wang [25] demonstrated that even if the local contact stresses interesting the PE asperities were higher than the nominal ones, the polymer deformation remained mainly elastic. Therefore the hypothesis of surface plastic deformation, hence the linear relation between the load and the real contact area at the basis of the Archard wear law, was no longer valid. Moving from this observation, a new formulation of the wear law for SoH contacts has been recently proposed by Liu et al. [26]. It proportionally relates the volumetric wear to the product of the nominal contact area \( A \) and the sliding distance, according to the following equation

\[ V = k_c A s \]  

where \( k_c \) is a dimensionless proportionality constant known as wear coefficient. The local instantaneous form of Eq. 11 becomes

\[ h_s(P, t) = k_c(P, t) |v(P, t)| \]  

Also \( k_c \) depends on the cross-shear ratio; the expression of \( k_c \) proposed by Liu et al. [26], i.e.

\[ k_c(CS_s) = \begin{cases} (32CS_s + 0.3) \times 10^{-9} & 0 \leq CS_s \leq 0.04 \\ (1.9CS_s + 1.6) \times 10^{-9} & 0.04 \leq CS_s \leq 0.5 \end{cases} \]  

was obtained from the \( k_f \) defined in [30]. It is worth underlying that \( k_c \) is assumed to be almost constant with the contact pressure.

All the above mentioned wear laws and wear factor/coefficient expressions are collected in Table I together with the references in which they were exploited and defined. The graphical representations of the \( k_f \) and \( k_c \) expressions are shown in Fig. 2 to better clarify the dependence of such functions on the \( CS_s \).

### 3. Wear prediction model

An analytical and parametric wear predictive model was developed by the authors for SoH hip implants. The software Mathematica® was chosen for implementation as it is a very powerful tool for symbolic calculus.

The model was formulated for a left hip implant, characterized by a plastic cup coupled with a metallic or ceramic head, as represented in Fig. 3-a.

#### 3.1. Basic assumptions

The wear model was based on some hypotheses largely adopted in the literature and reported in Sec. 2. The following additional assumptions were made with the aim of simplifying the model and reducing the computational cost:

1. the geometrical variation of the plastic surface due to wear does not affect the contact mechanics;
2. the contribution of the elastic deformation of the cup to the sliding velocity can be disregarded;
3. the contact is considered frictionless;
4. the creep effects are neglected.

Particular attention has to be given to the first hypothesis that can be considered valid if the wear model is used for a comparative analysis of the wear laws and not for corroborating experimental findings.
### Table 1: Analytical expressions of wear factors/coefficient

<table>
<thead>
<tr>
<th>Wear law</th>
<th>k_f / k_c expression</th>
<th>Ref.</th>
<th>Type</th>
<th>Notes</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_f L_{ss}</td>
<td>k_f cost</td>
<td>1.066</td>
<td>Maxian et al. 1996 [14,15]</td>
<td>PoD</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>k_f (R_n)</td>
<td>8.68 \times 10^{-6} R_n + 1.51 \times 10^{-6}</td>
<td>Raimondi et al. 2001 [16]</td>
<td>HS</td>
<td>R_n (µm)</td>
</tr>
<tr>
<td></td>
<td>k_f (C_S)</td>
<td>3.28 \times 10^{-5} \ln(C_S) + 1.62 \times 10^{-6}</td>
<td>Kang et al. 2008 [23]</td>
<td>MD-PoP</td>
<td>Evaluated at 1 MPa</td>
</tr>
<tr>
<td></td>
<td>k_f (C_S, \bar{p})</td>
<td>\exp(-13.1+0.19\ln(C_S)-0.29 \bar{p})</td>
<td>Kang et al. 2009 [24]</td>
<td>MD-PoP</td>
<td>--</td>
</tr>
</tbody>
</table>
| k_f A_s | k_f (C_S) | \begin{cases} 
(32 C_S^2 + 0.3)10^{-9} C_S \leq 0.04 & \\
(1.9 C_S^2 + 1.6)10^{-9} 0.04 < C_S \leq 0.5 & 
\end{cases} | Liu et al. 2011 [26] | MD-PoP | Calculated as k_f (C_S, \bar{p}) with k_f evaluated at 3.18 MPa | Kang et al. 2008 [30] |

Legend: PoD - Pin on Disk wear test, HS - Hip Simulator wear test, MD-PoP - Multi Directional Pin on Plate.

![Geometry materials & frames](image-url)

Figure 3: Geometrical characteristics of the model (a). Coordinate reference frames used for model implementation: C_F = [O_c, x, y, z] cartesian fixed frame oriented along the anatomical directions (z vertical, x anterior) and C_c = [O_c, x_c, y_c, z_c] cartesian frame fixed to the cup (b,c).

As the friction coefficient is very low (i.e. ≤0.065 [31]), the assumption of a frictionless contact was introduced, as in [16], because it implies that the reaction force between the head and the cup lies in the direction joining their centers and this allows an immediate identification of the theoretical contact point. Moreover the variation of the contact actions can be considered negligible. The creep effect was disregarded as not meaningful to the aim of this study, that is the comparison of the wear models. In fact, according to Willing and Kim [32] and Abdelgaied et al. [33], the creep term depends on the material, on the average pressure, on the thickness and on time. As geometry is not updated, cup thickness and pressure distributions remain the same for all the implemented wear models. Therefore considering the creep effect would mean to introduce the same additional term to the different wear depths and thus it was preferred to compare the latter directly. However, according to Abdelgaied et al. [33], the creep contributes for about 5% to the cup thickness reduction.

#### 3.2. Model input

##### 3.2.1. Geometry and materials

The geometry of the implant was characterized by head and cup radii, r_h and r_c respectively, and by the cup thickness t_c (Fig. 3-a). The position of the cup in the pelvic bone was specified by the anteverision (α) and the inclination (β) angles (Fig. 3-b). Two coordinate systems were introduced, described in Figs. 3-b,c: the cartesian fixed global frame C_F = [O_c, x, y, z] (with z in the vertical direction, x in the antero-posterior direction) and the cartesian fixed cup frame C_c = [O_c, x_c, y_c, z_c].

The geometrical characteristics and the material properties of SoH simulated implant are summarized in Table 2.

##### 3.2.2. Boundary conditions

The cup was considered to be fixed in the pelvic bone and the relative motion assigned to the head. Consequently, the boundary conditions (BCs) consisted in the load applied to the head and the head angular velocity, both conveniently expressed in the reference frame C_F. In this study the wear caused by walking was investigated, thus the loading and the kinematic conditions of the gait cycle were implemented according to liter-
Table 2: Geometrical and material data of the simulated SoH hip implant. $E$ and $\nu$ are the Young modulus and the Poisson ratio for head and cup material, subscripts h/c respectively.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_c$ (mm)</td>
<td>Cup UHMWPE $E_c$ (GPa) 0.5</td>
</tr>
<tr>
<td>$c-h-r_c$ (mm)</td>
<td>$\nu_c$ 0.4</td>
</tr>
<tr>
<td>$t_c$ (mm)</td>
<td>Head CoCr $E_h$ (GPa) 210</td>
</tr>
<tr>
<td>$R_c$ (µm)</td>
<td>$\nu_h$ 0.3</td>
</tr>
<tr>
<td>$\alpha$ (°)</td>
<td></td>
</tr>
<tr>
<td>$\beta$ (°)</td>
<td></td>
</tr>
</tbody>
</table>

3.3. Wear evaluation

Both the Archard wear law and the new wear law were implemented in the local instantaneous forms, Eq. 2 and Eq. 12, respectively. All the expressions of $k_f$ and $k_r$ reported in Table 1 were taken into account. It is worth noting that the simplified cross-shear ratio $CS_y$, defined in Eq. 8, was employed.

The wear simulation went through three steps concerning the evaluation of the contact pressure, of the sliding distance and of the wear factor/coefficient. The latter, in particular the calculation of $CS_y(P)$, resulted the most critical point of the model, because it required the solution of the minimization problem that can be computationally expensive.

3.3.1. Contact analysis

Contact analysis was a fundamental part of the wear model for evaluating the contact area and the contact pressure. Usually analytical models, e.g. in Raimondi et al. [16], are based on the Hertzian theory, although it is hardly applicable in case of conformal geometry and of SoH bearings.

In this study the contact mechanics was solved by means of FE analyses and results fitted in order to be included in Mathematica® as continuous functions.

**FE analyses**

A simple axial-symmetric FE model was developed in Ansys®, in parametric form for simulating several geometries and load levels. The external side of the cup was considered fixed and the load applied to head as a pressure distributed on its midside surface. A frictionless contact pair was defined between the external surface of the head and the inner surface of the cup. Static analyses were performed increasing the load from zero up to 3000 N in steps of 100 N. A mesh sensitivity analysis was carried out to define the contact element size (1 element/1° angle at the head center), which gave a variation on the maximum pressure below 0.2% with respect to a further mesh refinement (1 element/0.75°).

**Fitting of FE results**

By fitting the FE results (for a given hip implant), an analytical correlation was established between pressure response and two contact parameters, the maximum contact pressure $p_{max}$ and the semi-angular contact width $\psi_{max}$ (Fig. 6-a)

$$p(\psi) = p_{max} \left[ 1 - \left(\frac{\sin \psi}{\sin \psi_{max}}\right)^c\right]$$

where $\psi$ is the angle between the loading direction and the radius of a point $P$ of the cup surface, and $c$ is the power law exponent, calculated from the equilibrium condition. It should be noted that $p_{max}$, $\psi_{max}$ and $c$ depend on the load. Their trends for the simulated MoP implant (Tab. 2) are plotted in Fig. 6.

3.3.2. Sliding velocity

The sliding velocity is the velocity of a point $P$ of the cup surface with respect to the corresponding contact point $Q$ on the head. Considering the cup as fixed and neglecting the effect of the cup elastic deformation on the kinematics, the sliding velocity could be easily evaluated as the opposite of the velocity of $Q$

$$v_P = -v_Q = -(v_{Oh} + \omega \times O_h Q)$$

where $O_h$ is the head centre and $\omega$ the head angular velocity. The position of $O_h$ was calculated knowing the instantaneous loading direction.
3.3.3. Wear indicators

The proposed model evaluates the linear wear depth $h(P)$ at every point of the cup surface

$$ h(P) = \int_0^T h_c(P, t) \, dt $$

(16)

and the volumetric wear $V$

$$ V = \int_A h(P) \, dA $$

(17)

over a single gait cycle. However, since the up-date of the geometry was not implemented, the wear after $n$ gait cycles was obtained simply by multiplying $h$ and $V$ by $n$.

4. Results

All the results presented in this section were obtained for the MoP hip implant characterized in Tab. 2, having $r_e = 14$ mm, $cl = 0.08$ mm, $t_e = 8$ mm, $\alpha = 0^\circ$ and $\beta = 35^\circ/45^\circ$ for the in-vivo/hip simulator BCs respectively.

Wear indicators, i.e. the linear and the volumetric wear, are referred to $10^6$ cycles (1 Mc), that is approximately 1 year of activity [35]. Moreover pressure, $CS$, and linear wear maps are visualized by means of 2D contour plots, projected on the $x_c - y_c$ plane. It is worth reminding that it was assumed that only the acetabular cup is subjected to wear, therefore no results are reported for the head.

![Figure 6: Contact parameters of Eq. 14: (a) schematic representation; (b) trends vs. load.](image)

![Figure 7: Trajectory of the theoretical contact point (a) and contact pressure distribution at the maximum load (b), in the $x_c - y_c$ plane under in-vivo gait conditions.](image)

4.1. Comparison of the wear laws

The comparison of the wear laws in Table 1 was carried out assuming in-vivo gait conditions, i.e. Figs. 4-c,d. In all cases, the trajectory of the theoretical contact point $P_c(t)$ and the contact pressure at the maximum load are those depicted in Figs. 7-a and b, respectively. The linear wear maps predicted by the proposed model according to different wear laws are shown in Fig. 8. All the maps are characterized by a worn area located laterally to the north pole of the acetabular cup and extended to both the anterior and the posterior quadrants. Furthermore all the wear laws indicated a maximum wear depth nearly in the same cup region, where the trajectory of $P_c(t)$ and the maximum contact pressure occur. A qualitative analysis of Fig. 8 also points out that similar wear maps are predicted by the models based on the Archard’s wear law, though varying the wear factor expression, whilst a different wear distribution was obtained using the new wear law, (i.e. Eq.11). Indeed in the latter case the maximum linear wear affected a wider area. On the other hand, significant differences arise from a quantitative examination of results shown in Fig. 8: both the linear and the volumetric wear are strongly affected by the particular wear factor/coefficient used. The resulting maximum wear depth and the volumetric wear varied in wide ranges, 0.035–0.158 mm and 12–37 mm³ respectively, both assuming the highest value for $k_i(R_d)$ and the lowest value for $k_i(CS_r)$.

4.2. Effect of the kinematic conditions

The effect of the kinematic conditions on wear was investigated by comparing the wear predictions associated to three different motion types, characterized by one, two and three non-zero angular velocity components (Figs. 4 b), respectively. In particular the former case simulates a simple flexion-extension motion, assuming only $\omega_z$ different from zero; the second one, labelled $\omega_x - \omega_z$, add the internal-external rotation about the $z$-axis while the latter case, with full gait three-dimensional vector, is denoted as $\omega_x - \omega_y - \omega_z$. Additionally, for each velocity component, the temporal trend of in-vivo gait cycle (Fig. 4 d) was assumed. Each kinematic condition caused different sliding velocities and therefore also different $CS_r$. For the three cases, the obtained average $CS_r$ was 0.172, 0.034 and 0 decreasing with the number of (non-zero) angular velocity com...
component. Indeed under the only flexion-extension movement the sliding velocity had a constant direction and hence $CS_x = 0$. Figures 9-a,b show the $CS_x$ maps for the other two cases, which appear significantly different. In fact, in the full 3D condition (Fig. 9-a) the central region of the worn area (close to $x_c = 0$) is characterized by high values of $CS_x$, whilst its posterior/anterior edges by low ones. An opposite trend can be observed for the $\omega_x - \omega_z$ condition: Fig. 9-b clearly shows an almost zero $CS_x$, for the contact points close to $x_c = 0$, which in fact have nearly uni-directional trajectories. The wear indicators obtained with the three cases are summarized in Fig. 10, showing the influence of the kinematic conditions; the more complicated and realistic the angular velocity, the higher $CS_x$ and the higher the linear and the volumetric wear rates. The wear depths varied with the wear law confirming trends observed in Sec. 4.1: the highest and the lowest wear depths were obtained for $k_f(R_z)$ and $k_f(CS_x)$, respectively. As shown in Fig.10-c, the percentage difference of $V$ and $h_{max}$ between complete 3D and simple flexion-extension movement ranged between 16 – 44% and 17 – 45%, respectively. This percentage variation highlighted the higher sensitivity of the model based on $k_f(CS_x)$ to the kinematic conditions, which is significantly reduced when also the mean pressure is taken into account in $k_f(CS_x, \bar{p})$.

4.3. Effect of the loading conditions

The effect of the loading conditions was investigated in two directions: on the one hand as effect of one or more load components and on the other hand as influence of the load level, which is related to the Body Weight (BW). In both cases, the kinematic conditions were those of the 3D in-vivo gait (Fig. 4-d), therefore the $CS_x$ map corresponds to Fig. 9-a.

In analogy with the previous section, the action of $L_z$ alone or combined with the other load components (Fig. 4-a) was considered, assuming the trends of the in-vivo gait cycle in Fig. 4-c. Since the vertical component ($L_z$) is much higher than the other ones, the simulations produced very similar results, corresponding to those reported in Sec.4.1. It is worth noting that the presence of time-varying components gives a variable load direction and therefore a curve of nominal contact points on the cup (Fig. 7-a); on the other side a single component entails a fixed contact point.

Then, the effect of the Body Weight was investigated by comparing the wear induced by three different loads levels obtained scaling the in-vivo load in Fig. 4-c of 80-100-120%. The resulting wear indicators are reported in Fig. 11. It can be observed that all models based on the Archard wear law show a rather linear variation of both the linear and the volumetric wear with load. Differently, assuming the new wear law, the volumetric wear slightly increases by increasing the load, although the linear wear depth remains unchanged. In particular, a load increase (decrease) by 20% induces differences in $h_{max}$, and $V$ of $0 – 13\% (-15 – 0.6\%)$ and $9 – 21\% (-20 – -9\%)$ as detailed in Fig. 11-c, respectively. The variation of the wear indicators according to the wear law was in agreement with those of Figs. 8 and 10: the highest $h$ and $V$ values were obtained for $k_f(R_z)$ and the lowest ones for $k_f(CS_x)$.

4.4. Comparison of the wear predicted under in-vivo and hip simulator conditions

The combined effect of the kinematic and the loading conditions on wear was finally investigated, comparing the in-vivo and the hip simulator boundary conditions, reported in Figs. 4-c,d and Figs. 4-e,f respectively.

First of all it is worth noting that the two conditions caused different contact pressures and $CS_x$ values. The contact pressure due to simulator loading was higher up to 33% compared

![Figure 8: Acetabular cup linear wear maps (in the $x_c - y_c$ plane) and volumetric wear after 1 Mc, predicted using different wear laws, under in-vivo gait conditions. In the first row each plot is represented within its own minimum and maximum values, while in second row the same range is used for all plots to ease their comparison.](image-url)
to the *in-vivo* conditions. The average \( CS_s \) value under simulator BCs was 0.088, lower (-50%) than the *in-vivo* one. The \( CS_s \) maps for the two cases are shown Figs. 9-a, c for the *in-vivo* and hip simulator kinematics, respectively. For the latter, the \( CS_s \) lowest values were predicted close to the \( x \) axis, whilst the highest ones in the posterior and anterior edges of the worn area. The map was similar to that one of \( \omega_x - \omega_y \) case (Fig. 9-b), which indeed accounted for the same motion components of the simulator.

As far as wear results are concerned, different wear maps were predicted for the two cases. As an example, those obtained both for the Archard wear law with \( k_f(CS_s, p) \) and for the new wear law are compared in Fig. 12. It can be observed that both BCs gave similar maps when the Archard’s wear law was used (Fig. 8-d vs. Fig. 12-a), although with different maximum values (0.066 mm vs. 0.055 mm), higher for the *in-vivo* case. On the contrary the new wear law produced rather different maps (Fig. 8-e vs. Fig. 12-b) and maximum wear depth for the simulator condition (0.035 mm vs. 0.037 mm). A comparison of the maximum linear wear and the volumetric wear under the *in-vivo* and the simulator BCs for all wear models is proposed in Fig. 13: both the wear indicators resulted lower under simulator BCs, with the exception of \( h_{max} \) calculated assuming \( k_c(CS_s) \), as commented above.

Figure 9: \( CS_s \) maps for three different kinematics conditions: complete *in-vivo* gait velocity (a), simplified case with \( \omega_x - \omega_y \) (b), Leeds ProSim simulator (c). In the left column each plot is represented within its own minimum and maximum values while in the right column all the \( CS \) plots are depicted in the same range (0–0.5).

Figure 10: Effect of the kinematic conditions on the linear (a) and the volumetric (b) wear after 1 Mc. Percentage volumetric wear variation between a full 3D and a flexion-extension motion (c).

5. Discussion

The comparison of the most recent wear laws for SoH hip implants, implemented in the proposed wear model, underlined a remarkable variability of the results with the wear factor/coefficient. The sensitivity of the wear laws to the kinematic and the loading conditions is discussed in the following sections. Some general remarks on CS ratio definition and evaluation are also reported.

5.1. Effect of the kinematic conditions

The kinematic conditions affect the wear law through both the sliding velocity and the \( CS_s \). Accordingly, our results were influenced by the kinematic conditions, especially those obtained with wear laws dependent on the \( CS_s \). In particular, it was observed that the more complicated the kinematic conditions, the higher the \( CS_s \) and the wear factor/coefficient, the higher the wear indicators, (the variations of \( h_{max} \) and \( V \) were similar). Indeed, as shown in Fig. 10, the full gait motion caused higher \( V \) and \( h_{max} \) than the simplified flexion-extension one and their percentage differences were maximum.
5.2. Effect of the loading conditions

Our model also allowed to investigate how the wear of hip implants is affected by the loading conditions and by the patient’s body weight. Results indicated that, for the in-vivo condition, a three-dimensional load produced similar wear as the uniaxial vertical one, for all the wear laws. Moreover a scale factor on the load induced a rather proportional wear volume and wear depth when the Archard wear law was considered; on the other hand, the new wear law predicted a V variation approximately half the load variation while the wear depth remained almost unchanged. Consequently, it can be affirmed that the two wear laws exhibited a different response to the load level. In fact the load affects the wear through the contact pressure and the contact area in the Archard wear law, but only through the contact area in the new wear law. This result suggested the new wear law as the most suitable one for describing the effect of the load on wear. Indeed experimental investigations have demonstrated a poor correlation between load and wear, in terms of both loading components and law. As an example, similar wear rates were predicted by hip simulators adopting a 3D load [39] versus a vertical load [40]. Other studies confirmed that simplified loading cycles produced wear rates similar to the physiological ones [41, 42]. As far as the loading conditions is concerned, the only aspect which might affect the wear is the velocity of the load application. The hip simulator study presented in [43] reported that, for plastic cup with high $R_a$ ($\approx 0.3 \, \mu m$), an increase in load speed led to a massive increase in wear.

Actually, the effect of the activity level on the wear still remains an open issue. The exposure of hip joints to a greater number of walking cycles or different activities should theoretically contribute to variation in the long-term wear performance of their replacements [44]. Nevertheless experimental studies showed no relationship between wear and patient activity levels [45–47]. Unfortunately, nowadays numerical investigations on this aspect cannot be carried out since they would require $k_f$ and $k_s$ being evaluated under a dynamic pressure rather than a constant one.
5.3. Comparison of the wear predicted under in-vivo and hip simulator conditions

Wear simulations are often carried out by means of experimental tests in hip simulators, which implement simplified kinematic and loading gait conditions. Our results support the necessity to design hip simulators that reproduce physiological gait conditions as much as possible especially in terms of kinematics, whilst a simplification can be tolerated on the loading components and magnitude, which is in agreement with [48, 42, 41]. In particular our results indicated that under simulator conditions the wear rates were underestimated with respect to the in-vivo conditions. The underestimations of V and $h_{\text{max}}$ varied with the wear law, however being quite high, up to 28% and 44%, respectively. Nevertheless it should be considered that many different hip simulators are actually used to test wear of hip implants. As reviewed by Calonius and Saikko [48], such simulators apply different kinematics which generate different slide tracks and likely cause different wear.

5.4. Remarks on the definition and the calculation of the CS ratios

The cross-shear effect is surely a fundamental aspect of the wear of SoH bearings, but unfortunately there are still some open issues on its modelling. According to Eq. (8), the $CS_f$ can be defined only for contact points where the sliding velocity is non-zero during the whole gait cycle (otherwise Eq. (8) would yield to an indeterminate case). However in some studies, as [23, 26], the $CS_s$ is evaluated all over the cup surface without considering where the contact actually occurs, as if the relative velocity was used instead of the sliding velocity in Eq. (8). An example of the $CS_s$ map obtained with the relative velocity for hip simulator BCs is reported in Fig. 14-a: the maximum $CS_s$ ratios were localized in the medial cup, outside the contact area, in agreement with the findings in [23]. Obviously the $CS_s$ maps evaluated according to the simplified and the full definition (Fig. 9-c) matched were the contact occurred.

Another critical point regards the use of the simplified form of the $CS_f$ (Eq. (8)), in particular the replacement of the instantaneous pressure with its average value. In order to verify the validity of this assumption, some simulations were carried out calculating the $CS_f$ according to Eq. (7). As an example, the maps of $CS_f$ obtained under hip simulator and in-vivo BCs are shown in Figs. 14-b,c, respectively, which differ from the correspondent $CS_s$ ones shown in Figs. 9-c,a. Such difference is more evident in the case of in-vivo BCs since the area with non-zero $CS_f$ appears like moved towards the anterior direction with respect to the $CS_s$ map. The average (maximum) values of cross-shear ratio comparing $CS_f$ vs. $CS_s$ gave 0.09 vs. 0.1 (0.31 vs 0.23) and 0.17 vs. 0.15 (0.48 vs. 0.4) for hip simulator and in-vivo BCs, respectively. However, despite such differences, rather negligible variations were observed in the wear.
5.5. Comparison with predicted and clinical wear estimations

The current model was developed with the aim of comparing the most recent wear laws for the UHMWPE acetabular components. Indeed such laws were implemented in models with different characteristics (e.g. geometries, material properties or BCs) and hence not directly comparable. For the same reason the comparison of our results with other numerical wear rates can be only qualitative and it is certainly even harder with respect to clinical wear rates, typically highly dispersed. Nevertheless, on completion of the discussion section, a comparison with the literature is provided in the following.

As far as the numerical wear predictions is concerned, slightly lower wear rates were predicted by our model. Raimondi et al. [16] investigated an implant with a higher clearance (cL=0.1 mm vs. cL=0.08 mm) and obtained V=70 mm³/Mc and about hmax=0.85 mm/Mc under in-vivo loading conditions from [53] with BW 70 kg and kinematic conditions from [54]. Kang et al. [23], for an implant with same radius as our but clearance not specified, predicted V=26.7 mm³/Mc and hmax=0.07 mm/Mc under Leeds ProSim hip simulator BCs (kT for average pressure of 1 MPa and for 0 MRad polyethylene, according to Table 1). Kang et al. [24] estimated V=13 mm³/Mc and hmax=0.035 mm/Mc for a MoP implant similar to [16], under Leeds ProSim hip simulator BCs. It is worth noting that the contact pressure was calculated according to the hertzian theory in [16] and to the constrained column model in [23, 24]. Liu et al. [26] assumed an MoP implant with the same geometry of the proposed one, under Leeds ProSim hip simulator BCs.

Indicators, e.g. the percentage difference of V and hmax ranged between 0 – 2%. Consequently, also considering the higher computational cost of the CSf calculation with respect to CSs, the CSs can be considered an acceptable simplification.

It should be noted also that the use of the correct definition of the CSs in the new wear law would cause the wear rate to be explicitly dependent on the contact pressure, which is in disagreement with the law itself, formulated ad hoc for eliminating such dependence. A further clarification is needed as far as the new wear law is concerned. The expression of k_c proposed by Liu et al. [26] (Eq. (13)) was obtained from the k_f defined in [30] according to k_c(CS_s) = k_f(CS_s) p, where p is the constant pressure used in the wear test. Additionally Liu et al. [26] considered k_c almost constant under the pressures of interest, although a significant variation is showed in their study, particularly in the range 1 – 6 MPa. Indeed the calculation of k_c as k_c(CS_s, p) = k_f(CS_s, p) p, exploiting data set from [24], would yield to the dependence of k_c with the pressure plotted in Fig. 15-a.

Another questionable point concerns the analytical expressions of k_f(CS_s), k_f(CS_s, p) and k_c(CS_s) obtained by pin on plate wear tests under constant pressures. Firstly, the experimental test campaign was quite limited and a scant number of data was used for extrapolating k_f and k_c curves. Thus the fitting of the same data set by different functions yielded to rather different trends, as depicted in Fig. 15-b.

In addition sometimes the analytical expressions are not valid throughout the domain of CSs; e.g. in case of unidirectional motion (i.e. CS_s = 0) k_f(CS_s) provides an unreasonable value of the wear factor whilst k_f(CS_s, p) predicts no wear. In order to avoid this, a minimum CS_s value equal to 0.06 was employed in the evaluation of the wear factor/coefficient. Finally the effect of a dynamic pressure on the wear factor/coefficient was neglected even if it is recognized to strongly affect the wear of UHMWPE [49].

Beyond the limitations discussed thus far, a recent study by Lee et al. [50] has highlighted the necessity of a better understanding of the CS effect. In their attempt to clarify some inconsistencies between theoretical and experimental findings, Lee et al. [50] revised the CS_s definition previously proposed by the same group (Eq. (3)). They basically observed that the frictional work released in the PMO and the CS direction, concurred both to molecular reorientation and to wear. However Lee et al. [50] suggest not to extrapolate their conclusions and consequently their indications were not included in the proposed wear model.

Finally, other wear models of the cross-shear effect are worth mentioning, as the one proposed by Hamilton et al. [51] for total knee replacement and the more recent one by Dressler et al. [52], which takes into account the crossing motion history and the actual path scale. Such models were not considered in this study as they were not yet applied to hip replacements and this would require specific validation tests.
with a modified loading cycle [55], obtaining $V = 23 \text{ mm}^3/\text{Mc}$ and about $h_{\text{max}} = 0.042 \text{ mm/Mc}$. Their model, differently from the other ones, considered an elastic-plastic behavior for the plastic cup.

The current wear model, as the above mentioned ones, underestimates clinical wear measured both in-vivo and ex-vivo.

For instance, Kabo et al. [4] evaluated an average $V$ rate of 75.6 mm$^3$/Mc and average linear wear rates of 0.234 mm/Mc for implants with $r_h = 14 \text{ mm}$. Livermore et al. [3] reported a volumetric wear rate in the range 0 – 225 mm$^3$/Mc (average value of 48.4 mm$^3$/Mc) and linear wear rates in the range 0 – 0.3 mm/Mc (average value of 0.08 mm/Mc). More recently, much higher wear rates were measured by Chuter et al. [10] of about 105 – 430 mm$^3$/Mc (average value of 227 mm$^3$/Mc) and 0.21 – 0.66 mm/Mc (average value of 0.45 mm/Mc). However it should be considered that a comparison with clinical data is very difficult because of their high dispersion mainly due to dependence of in-vivo wear on many factors (e.g. BW, age, lifestyle/daily activities, PE oxidation and aging). Moreover it has been demonstrated that the wear measurements can also be affected on the experimental method exploited [10].

However the deviation between our results and numerical/clinical wear rates reported in the literature was expected as a consequence of the model simplifying hypotheses which were necessary for the development of a numerical tool with a low computational cost.

6. Conclusion

An advanced mathematical wear model for SoH hip implants was developed. Several aspects were improved with respect to the previous mathematical wear model by Raimondi et al. [16], among which: the contact pressure, calculated by FE analysis instead of Hertzian theory; the sliding distance, evaluated more precisely by applying the kinematic law rather than by summing circumference arches; the wear law, since the present model implemented the most recent wear laws including the cross-shear effect. The parametric formulation and the low computational cost of our model were exploited to carry out a numerical comparison of such wear laws and to investigate their sensitivity to loading and kinematic conditions. All the simulations highlighted the remarkable variability of results with the wear law/factor. To give an example, under in-vivo gait conditions, percentage differences of $h_{\text{max}}$ and $V$ up to 210% and 350% respectively, were obtained (Fig. 8). As a general trend, the highest wear was predicted by $k_f(R_{sh})$; similar wear rates were obtained for constant $k_f$ and $k_f(C_S)$, whilst the lowest wear rates were for $k_f(C_S, p)$ and $k_f(C_S)$. In terms of relative variations, the wear model based on the $k_f(C_S)$ resulted the most sensitive to the kinematic and loading conditions. The fundamental role of the wear law/factor in the reliability of the wear model was highlighted, as well as some critical aspects of the examined laws. Ongoing research will address the improvement of the model by implementing the up-date of the geometry due to wear and by evaluating the component of the sliding velocity due to cup elastic deformation.


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Research Highlights

of the manuscript entitled

A comparative study of wear laws for soft-on-hard hip implants using a mathematical wear model

- A mathematical, parametric wear model for soft-on-hard hip implants was developed in Mathematica.
- The most recent UHMWPE wear laws, based on the cross-shear effect, were implemented and compared.
- Simulations highlighted the variability of wear predictions with the wear factors/laws.
- A sensitivity analysis showed that kinematic conditions affect wear more than loading ones.
- Despite some open issues on its evaluation, the main role of the cross-shear effect on wear comes out.