High-Accuracy Ring Laser Gyroscopes:  
Earth Rotation Rate and Relativistic Effects

N Beverini 1,2,6, A Di Virgilio 2, J Belfi 2, A Ortolan 3, K U Schreiber 4, A Gebauer 4 and T Klügel 5
1 Department of Physics, Università di Pisa, Largo Pontecorvo 3, I 56127 Pisa, Italy
2 INFN, Sezione di Pisa, Largo Pontecorvo 3, I 56127 Pisa, Italy
3 INFN, Laboratorio Nazionale di Legnaro, I 35020 Legnaro, Italy
4 Forschungseinrichtung Satellitengeodäsie, Techn. Universität München, Germany
5 Bundesamt für Kartographie und Geodäsie, GO Wettzell, Germany

e-mail: nicolo.beverini@unipi.it

Abstract. The Gross Ring G is a square ring laser gyroscope, built as a monolithic Zerodur structure with 4 m length on all sides. It has demonstrated that a large ring laser provides a sensitivity high enough to measure the rotational rate of the Earth with a high precision of $\Delta \Omega < 10^{-8}$. It is possible to show that further improvement in accuracy could allow the observation of the metric frame dragging, produced by the Earth rotating mass (Lense-Thirring effect), as predicted by General Relativity. Furthermore, it can provide a local measurement of the Earth rotational rate with a sensitivity near to that provided by the international system IERS. The GINGER project is intending to take this level of sensitivity further and to improve the accuracy and the long-term stability. A monolithic structure similar to the G ring laser is not available for GINGER. Therefore the preliminary goal is the demonstration of the feasibility of a larger gyroscope structure, where the mechanical stability is obtained through an active control of the geometry. A prototype moderate size gyroscope (GP-2) has been set up in Pisa in order to test this active control of the ring geometry, while a second structure (GINGERino) has been installed inside the Gran Sasso underground laboratory in order to investigate the properties of a deep underground laboratory in view of an installation of a future GINGER apparatus. The preliminary data on these two latter instruments are presented.

1. Introduction
Ring laser gyroscopes (RLG) have demonstrated to currently be the most sensitive device for testing rotational motion with respect to an inertial frame. A new era for upscaled ring laser gyroscopes started in the 90’s of the past century when, thanks to the technological improvements in the production of low loss mirrors, a reflectivity exceeding 99.99% was achieved. Unlocked Earth rotation sensing with a ring laser of about 1 m² of area was demonstrated at the University of Canterbury in Christchurch, New Zealand [1]. Subsequent further development led to the construction of the Gross-Ring (G), a 16 m² area ring laser, located at the Geodetic Observatory in Wettzell (Bavaria) [2]. G has continuously improved over the years, by more and more accurate control of the environmental

6 To whom any correspondence should be addressed
conditions, finally achieving a rotational sensitivity of the order of 0.25 p-rad/s over 10^4 s, which corresponds to resolving the Earth rotation rate to 3 part in 10^9.

In the VIRGO laboratory in Cascina near Pisa (Italy) a moderate size ring laser of 1.82 m² area dubbed G-Pisa was realized by a collaboration between INFN and Department of Physics of the University of Pisa with the support of the expertise of Prof. Schreiber (Technische Universität München (TUM)) in 2008. The effort was inspired by the desire to establish very small ground tilt movements for the VIRGO interferometer. G-Pisa was first operated with the optical cavity orientated in the local horizontal plane and later in the vertical plane [3]. It was operating on March 11th, 2011, when it registered the rotation induced by the seismic Rayleigh waves generated by Fukushima earthquake [4]. Putting together the expertise acquired by the INFN group and that of the other principal groups operating large laser gyro, (TUM and University of Canterbury), a proposal was published in 2011 [5] that suggests a ring laser array with the accuracy needed to observe the Lense-Thirring effect. According to the theory of General Relativity, this effect arises from the metric deformation induced by a massive rotating body. Einstein’s equations predicts a 1 ppb correction to the Earth rotational rate, when measured in a laboratory frame bound to the Earth surface with respect to the rate defined by quasar positions measured by the Very Long Baseline Interferometry (VLBI). So far this frame dragging effect has been confirmed in the context of dynamic satellite experiments only [6, 7]. A “static” measurement in a terrestrial laboratory would give local information as opposed to an averaged measurement as it is provided by the satellite experiments. While the satellite measurements have confirmed the frame dragging effect, they have not yet reached a resolution that would put this theory to a test. In order to provide an improvement, a viable ring laser system would have to improve in sensitivity and above all in the long-term stability. Furthermore it is required to measure the rotation rate of the Earth absolute with sufficient accuracy, since the frame dragging effect appears like a small bias on top of the rate of rotation of the Earth itself.

2. Earth Rotation Rate and Relativistic Effects

2.1. Geophysical Signal Contributions
Large Ring lasers firmly tied to the Earth crust represent the only technology in space geodesy that is directly linked to the instantaneous axis of rotation of the Earth. With a sensor sensitivity of 12 p-rad/s/sqrt(Hz) inferred from the cavity ring-down time, G is sensitive to geophysical signals like polar motion, solid Earth tides, the Annual and the Chandler wobble and local tilts as small as 1 n-rad. A typical measurement sequence is shown in figure 1. Since laser gyro can operate continuously, they provide a continuous set of measurements, which is not yet available for VLBI. However, a good agreement between the VLBI and ring laser technology has been achieved by measuring and comparing the low frequency Chandler- and Annual-Wobble, a free oscillation of the Earth [8]. The remaining discrepancy is due to the fact that local tilts of the single component G ring laser currently cannot be measured with a sufficient long-term stability due to small and slow drift effects in the respective tilt meters. This is work in progress. A moderate improvement in sensitivity and a full 3-dimensional detection of the Earth rotational velocity vector would take the ring lasers into a regime

![Figure 1: Fluctuation of the Earth rotational rate due to Diurnal polar motion and solid Earth tides, as observed by the laser gyroscope G at the Geodetic Observatory in Wettzell (Bavaria)](image)
where they can be efficiently integrated into the data produced by VLBI observations.

2.2. Relativistic Signal Contributions
In the weak-field approximation of Einstein’s equation for General Relativity, the expected rotation signal seen by a ring laser located in a laboratory on the Earth surface, with co-latitude $\theta$ and with the axis contained in the meridian plane at an angle $\psi$ with respect to the zenith, can be written as:

$$
\nu = 4 \frac{S}{\lambda p} \Omega_t \left[ \cos(\theta + \psi) - 2 \frac{GM_r}{c^2 R_t} \sin\theta \sin\psi + \frac{GI_z}{c^2 R_t^3} (2 \cos\theta \cos\psi + \sin\theta \sin\psi) \right],
$$

where $G$ is Newton’s gravitational constant, $\Omega_t = 7.29 \cdot 10^{-3}$ rad/s is the rotational velocity of the Earth, $M_r$ the mass of the Earth, $R_t$ the Earth mean radius, and $I_t \approx 2/5 M_r R_t^2$ the Earth moment of inertia. The first term in the square bracket corresponds to the classical Sagnac signal; the second one, radially oriented, is produced by the coupling of the Earth Newtonian field with the rotation of the laboratory carried by the rotating Earth (geodetic or De Sitter precession) and the third one, characterized by a dipolar structure, is produced by the rotating mass of the Earth and is proportional to the Earth angular momentum ($\text{Lense-Thirring precession, LT}$) [9]. Both these last two relativistic effects are reduced with respect to the classical Sagnac effect by a factor that is of the order of the magnitude of the ratio between $R_t$ and the Schwarzschild radius of the Earth $2GM_r/c^2$. This corresponds to values of around $10^{-9}$. These effects can be observed as a difference between the rotation rate observed by a ring laser in the rotating frame of the laboratory and the length of the day determined by the inertial frame of the “fixed stars” by IERS through the technique of VLBI.

3. GINGER project
The performance of G is near the shot noise limit and therefore defined by the properties of the mirrors. An impressive long-term stability of the order of 1 part in $10^6$ of the rotation rate of the Earth has been obtained by an accurate modeling of all the environmental effects of geodetic, geophysical, or meteorological origin. However the system lacks absolute accuracy: it is only sensitive to a single component of the angular velocity vector and the absolute orientation of the laser cavity with respect to the fixed stars inertial frame has not been established with the required degree of accuracy. To reach the goal, we need to improve the instrumental setup in several ways:

1. **Reduction of the shot noise threshold**: This can be obtained by enlarging the size of the sensor up to 7 - 9 m (following [8], for equivalent mirrors the sensitivity increases as $L^u$ with $u \approx 5$). A larger structure is not useful: this would increase the beam radius and then the mirror losses.

2. **Improvement of the long-term stability**: By means of monitoring and controlling the laser operative parameters and the environment the remaining drift can be reduced. A deep underground sensor location seems necessary to reduce surface tilts due to atmospheric load variation caused by changing weather patterns and local hydrological effects.

3. **Stabilization of the geometrical scale factor**: This requires a strict active stabilization control of the laser beam path geometry and an accurate orientation with respect to the Earth axis.

4. **Correction of the nonlinear laser dynamics**: Intracavity dynamics of the laser gain medium introduce nonreciprocal effects, resulting in a null-shift term in the gyroscope response. To achieve the required accuracy, an estimate of these contributions must be implemented [10].

The long-term stability of G is warranted by its rigid monolithic structure based on the near zero thermal expansion material Zerodur, but this alone is not enough. Small variations of the laboratory temperature and adiabatic compression and expansion with changing atmospheric pressure are causing small changes in the backscatter coupling of the two counter- propagating laser beams and hence a non negligible drift in the measured rotation rate. The present situation does not allow the construction of monolithic structures larger than G, because the necessary ultra-low expansion material is not available in the required size. A viable solution to this problem is the implementation of a closed loop active control system that stabilizes not only the perimeter length, as already done in G, but also the geometrical shape of the ring optical cavity and hence the mechanical scale factor $k = A/p$. In a near-
square 4-mirror cavity, the pairs of opposite mirrors define two Fabry-Perot cavities along the diagonals of the ring laser area. When the lengths of these resonators are kept locked to the same value, it is possible to demonstrate that in the square geometry the optical length of the perimeter is a stationary function of the position of the mirrors and consequently any deviation from the exact geometry gives only a second order contribution to $k$. This property gives also the opportunity to define a procedure in order to approach the correct configuration starting from a non-optimized one [11]. Apart from the scale factor, it must be considered that the RLG signal is also affected by the orientation of the planar ring cavity with respect to the Earth rotational axis. The required accuracy in the sensor orientation is at the level of 1 n-rad, clearly impossible to be achieved by a horizontal cavity in an underground experiment. A set of more (at least 3) equal ring lasers interferometrically referenced to each other, can however allow the reconstruction of the invariant modulus of the rotational velocity of the Earth independently from the knowledge of the absolute orientation. The comparison of this scalar value with VLBI measurements provided by the IERS would then show the LT effect. A 3-dimensional geometry based on a set of six shared mirrors in an octahedral configuration ideally defines three mutually orthogonally nested ring laser cavities and will give a strong internal constraint on the geometry when locking the three Fabry-Perot diagonal cavities to identical values.

4. Toward GINGER

4.1. General Considerations

Work is presently in progress, in order to demonstrate the feasibility of the GINGER concept and to develop and validate the necessary technologies. At first, a detailed model of the laser operation has been realised, which applies a Kalman filter in order to correct the frequency shift induced by the coupling between the counter-propagating beams. The application of the algorithm to the G-Pisa raw data provided a tenfold reduction of the systematic shift error as well as a tenfold reduction of the Allan deviation for times larger than 100 s [10]. Two different installations are presently under test. The first one, GP2, is located at the INFN laboratory of Pisa and is devoted to test the concept of stabilizing the diagonals of a square ring laser and to control of the ring geometry. The second one is an upscaled ring laser structure “GINGERino”, placed inside the underground laboratory of Gran Sasso, beneath 1000 m of rock. This ring laser is installed together with tilt meters and more traditional seismic equipment, in order to qualify the site in view of GINGER experiment. As stated before, the localization in a deep underground site will be a critical step to guarantee the necessary long-term stability, reducing the surface tilts due to atmospheric load variations caused by changing weather patterns and local hydrological effects.

4.2. GP2

In order to stabilize the diagonals of the square cavity, the ring laser structure with a length of 1.60 m

![Figure 2 – GP-2 gyroscope, located in the Pisa INFN Laboratory (color on line)](image)

![Figure 3 – GINGERino apparatus in the underground Gran Sasso INFN laboratories (color on line).](image)
on each side is mounted to a granite slab of 1 ton (figure 2). The plane of the granite is inclined by 47° to the North direction, aligning the cavity orthogonal to the rotational axis of the Earth. In this way, the sensitivity of the interferogram to tilts is minimized. An extension of the vacuum system along the diagonals provides the access to the diagonal Fabry-Pérot cavities. The corner boxes containing the mirrors can be moved by 6 PZT driven actuators to control the geometric parameters in a small range around the ideal shape. The system became operational in the last months. In a previous experiment we had verified the technique to realize an comparison of the absolute lengths and the frequency locking to a stabilized iodine reference He:Ne laser of two Fabry-Perot cavities by a triple-modulated Pound-Drever-Hall circuit [12]. The technique has now been applied to GP-2, and the simultaneous locking of the two diagonal optical cavities to the reference laser has been observed continuously for some hours. At the present stage the stability on the average length of each diagonal resonator in GP-2 reaches the level of 1 nm after hundred seconds of integration of the error signals in closed loop [13].

4.3. GINGERino

The GINGERino structure is the evolution of the previous G-Pisa laser gyroscope to a larger size instrument. It uses the same type of corner boxes of GP-2 that house the mirrors and the same type of mechanical levers for aligning the ring optical cavity. The resonator was mounted on a cross-shaped granite basis of 4 tons of weight (figure 3) and linked up by vacuum pipes in order to obtain a square optical cavity of 3.60 x 3.60 m². The granite basis is supported centrally by a monument made of concrete, which is firmly attached to the tunnel floor. A seismic station is located alongside the ring, equipped with two very broad band seismometer (a Guralp-360 and a Trillium-240, the latter linked up to the Italian National seismic array). A two-dimensional tilt meter completes the setup. It was operated from May 2015 onwards. In figure 4 we show a preliminary Allan deviation illustration of the rotational signal, compared with G data. Despite the fact that the mirrors presently mounted in the ring are old and no longer provide the best quality parameters, a level better than 2·10⁻⁶ is achieved in

![Figure 4](image_url) - Relative stability of the rotational signal (Allan deviation) from GINGERino and from G (color on line).

![Figure 5](image_url) - Record of the signal generated by the seismic event on date 17-06-2015 and time 12:51:38 UTC in Southern Mid Atlantic Ridge (magnitude 6.4). In black (left scale) is reported the rotational signal from GINGERino; in red (right scale) the horizontal seismometer signal [13]. (Color on line)
100 s. Some Earthquakes were registered during these preliminary runs. As an example, Figure 5 shows the record of the rotational Love wave signal generated by the seismic event on 17-06-2015 at 12:51:38 UTC in Southern Mid Atlantic Ridge (magnitude 6.4), together with the seismometer signal. Work on the analysis of the correlation of seismic rotational and translational signals generated by this and by other earthquakes is presently in progress [13].

5. Conclusions
The recent performance of the G ring laser demonstrates the possibility for the ring laser technology to access the resolution required for sensing relativistic effects on rotation. Two main critical points have to be tackled for achieving the required progress. The first is connected to the residual uncontrolled deformation and tilts of the optical cavity of the ring resonator. The second regards the excess of ground rotational motion typically induced by the Earth surface environment. In particular the very low frequency tilts and rotations of the sensor site are critical. Two experiments in Italy are running for the investigation of these two aspects, GP-2 and GINGERino. GP-2 ring laser is designed for the study of active methods for controlling the deformation of the cavity shape, exploiting the stabilization of the diagonal lengths and is currently achieving a resolution of the order of 1 part in 10^9. The performance of such an approach is ultimately limited by the frequency stability of the optical frequency reference and thus compatible with the geometrical stability required by GINGER. Preliminary experiments [14] provide promising results for the stability in the constrained length of the two resonators. The sensitivity of the geometrical scale factor against out of plane angle errors remains to be addressed. The GINGERino ring laser is investigating the local site stability inside the underground laboratory of the INFN in GranSasso. The ring laser stability is currently reaching a level of a few ppm at about 100 s and is limited the ring laser hardware, namely by the fluctuation laser parameters connected to the frequency pulling of a large amount of intra-cavity backscattered light. The present installation demonstrated a level of stability of the environmental parameters that is sufficient to operate the laser for several days with a drift of less than one longitudinal mode spacing.

We expect an important reduction of the back-scattering effect by a new set of high quality mirrors that have been commissioned and will be installed in the next months. Improvements in the long term stability are also expected by a better isolation of the laser chamber (pressure stabilization). A full implementation of the already established backscatter correction algorithm is the logical next step. Preparations for this are currently in progress.

References: